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ORIGINAL RESEARCH

Formal Issues of Trope-Only Theories of Universals

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Abstract
The paper discusses some formal difficulties concerning the theory of universals of Trope-Only ontologies, from which the formal theory of predication advanced by Trope-Only theorists seems to be irremediably affected. It is impossible to lay out a successful defense of a Trope-Only theory without Russellian types, but such types are ontologically inconsistent with tropes' nominalism. Historically, Tropists' first way to avoid the problem is appealing to the supervenience claim, which however fails on its terms and, thus, fails as a ground for a solution to the higher-order or 'type' problem. A later solution involves the invariance of primitive equivalence relations in order to make universals ontologically innocuous. However, I argue that this latter solution fails to meet the requirements imposed on an ontologically unbiased nominalist attitude. So, this paper discusses how Trope-Only theories alter standard formal moves in Nominalism, and also is interested in clarifying further the formal assumptions for these problems.

1 Introduction

The purpose of this paper is to assess the extent to which Tropes-Only ontology (TO)[1]—viz. one based on systems of disjoint particulars only—is consistent with the nominalist constraints of its theory of predication, namely those that restrict the intended ontology to first-order particulars (tropes).[2]

[1] In general, Trope theories are those metaphysics ontologically committed, uniquely or not, with tropes. See Rojek (2008) for an introductory account to such a variety.

[2] The paper relies on the assumption that typically characterizes the trend of formal ontology that there may be a fruitful interplay between logic and ontology (Cocchiarella 2007). This is not to be confused with that of a necessary correspondence (Smith 2005). After all, logic(s) give(s) us the formal structures of languages and theories (and their models).

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TO theories cannot work without assuming some primitive equivalence relation, namely some reflexive, symmetric and transitive relation among tropes, generating the nominalist counterpart of realist universals (properties).[3] Unlike realist universals, these relations are not supposed to commit TO-ists to some reality "over and above" that of tropes and, for this very reason, are usually called "internal"—as opposed to "external". Unlike external relations, internal relations among tropes hold necessarily just because the related tropes exist.[4]

There are two main approaches which differ in the strategy they provide to make "internal" equivalence relations successful. One assumes that equivalence relations *supervene* on tropes, meaning that they (allegedly) add nothing to the original TO-ontology while accounting for universals. Examples of this approach are earlier TOs that take (exact) resemblance as the internal relation, like Campbell (1990), Williams (1966) and Williams (1997). To such an approach, from now on dubbed as *Standard* TO (STO), it is possible to include also more recent works like Daly (1997), Maurin (2002) and Simons (2003) (for criticisms directed toward the suitability of (exact) resemblance, see Rojek (2008) and Hochberg (1992)).

A second path characterises those I dub *Invariant* TO (ITO), which assumes that internal relations are those that are simply *invariant*. This solution has the merit of getting rid of the reductive feature of supervenience in accounting for the non-committing nature of its primitive relations. Examples of this second approach to TO are Keinänen et al. (2018) and Keinänen et al. (2019). ITO diverges from STO also in the specific equivalence relation taken as primitive, i.e., the relation of 1 : 1 proportion—and that of order—among quantity tropes.

Actually, STO and ITO share the assumption of primitive equivalence relations as *invariant*, but they diverge in how they account for such an invariance: STO takes the primitive equivalence relation to be invariant because of its second-order nature. Thus, appealing to supervenience is necessary to make the second-order nature of the relation metaphysically non-committing, i.e., universals do not have causal powers over and above those of tropes. On the other hand, ITO presupposes such an invariance as the source of the first-order nature of its primitive equivalence relation, with the result of making the relation automatically non-committing. I argue that both moves are ineffective.

The present paper proceeds as follows. Section 2 is devoted to providing the essentials of TO. In particular, I will focus on its general predication theory (Sect. 2.1). In Sect. 3, I will sketch the general account of predication that fits the nominalist purpose of discharging higher-order commitments, together with a discussion of

[3] I do not consider here those approaches based on different (non-equivalence) classes, like those based on *inexact* resemblance or that of quantitative distance, because such relations, being *non-transitive*, cannot generate doubts about their ability to play the role of universals. See Rojek (2008).

[4] Maurin writes that a relation is internal if and only if "it depends for its existence on the existence of its relata" (Maurin 2010, pp. 321–322). So, following Denkel (1997), we should not have anything ontologically extra "over and above" the related terms. But, the very point at stake is how to account for such an existential dependence. Options about internality will be discussed here. Notice that, differently, Ehring's Trope theory takes the membership relation between a trope and its natural class (universal) as an external relation. For further considerations referring to internal-external relations see also Armstrong (2005); Robb (2005).

the presuppositions that allegedly ground this formal strategy (Sect. 3.1). In Sect. 4, instead, I will turn back to TO showing that the interplay between TO predication and the nominalist strategy raises a problem for TO-ists in formal ontology. As I prove, a formal contradiction arises from that interplay—as it were to be expected. Actually, that contradiction is arguably what motivate the development of TO into two main branches: STO and ITO. From thereafter, then, the paper will be addressed to the discussion of each branch and will be split into two comparable but (partially) autonomous sections.

I begin with the one that historically came first: STO (Sect. 5). According to STO that very contradiction and further related arguments may be made innocuous by appealing to the *supervenience claim*: primitive equivalence relations (universals) supervene on basic first-order tropes and, so, are metaphysically non-committing. STO-ists seem to claim that, despite of all the formal problems that may be involved in assuming primitive equivalence relations as the source of TO-predication, these are made metaphysically innocuous because of the supervenient nature of higher-order properties. Nevertheless, in Sect. 5.1, I show that things are not so easy, because the appeal to supervenience does not explain so much and, worst, the supervenience claim is false. Hence, a formal theory of supervenience begs the question of a minimal semantic strategy of the kind discussed in Sect. 4 to discharge the distinctive ontology of whichever conceded second-order talk. Sect. 5.2 will elucidate some further drawback of the specific equivalence relation assumed by STO.

Doubts about STO, together with the problematic features of (exact) resemblance, can be thought of as what pushes for the development of further and stronger strategies to restore the inner coherence of TO. So, Sect. 6 is devoted to introducing and expounding the inner advantage of ITO consisting in assuming an alternative account of the internality of the primitive equivalence relations that is based on their invariance. Nonetheless, in Sect. 6.1, I will argue that ITO's attempt to ground the ontological innocence of primitive equivalence relations on their logicality does work, but only at the surface. On a closer look, this latter attempt presupposes what it must prove. So, the accommodation of higher-order entities—that allegedly comes with primitive equivalence relations—in a single level ontology is, again, formally impossible from the outset.

2 Essentials of TO

Austere trope theories are nominalistic theories because of the very peculiar nature of tropes. Tropes were originally introduced to provide an alternative nominalist account to what I call here Classical Nominalism (CN)—namely, that class of ontologies that are existentially committed to concrete individuals only.[5] The peculiar

[5] CN, because of the central role of the notion of individual (concrete) object(s), is currently widely recognised as an empirically unsatisfactory ontological position. The apparent contrast with the progressive weakening of the particle-like or atomistic model of physical reality, towards a more exquisitely relational frameworks, such as that represented by the physics of the force fields (quantum field theory), makes CN simply empirically undesirable (French and Krause 2006; Seibt 2002). Such a con-

nature of tropes mixes two characteristic features that are generally considered exclusive of universals and concrete individuals: abstractness and particularity. Tropes are *abstract particulars*. As abstract entities, tropes are *qualities* (properties and, perhaps, relations); as particular entities, tropes have *singular* occurrence and *quantitative* character and, as such, they may be interpreted as having (causal) powers. Tropes are those cases of qualities—e.g., cases of shapes, of colors and so on—and those cases of quantities—e.g., cases of mass, of charge and so on— that are included in the specification of each concrete object of the world at each scale. So, unlike CN, concrete objects are no longer basic *and* fundamental within TO but, instead, are in some way *derivative*.[6]

Tropes are not postulated *ex post* and *ad hoc*. Instead, the rationale for their introduction is their plausibility as part of the ontological space. As Aristotle (Categories, 1a20-1b9) showed, there is nothing contradictory in thinking of abstract *and* particular entities. The evidence was their occurrence in what is often called "the ontological square" (Table 1): "a 2×2 matrix which results from the juxtaposition of concrete-abstract and particular-universal oppositions" (Rojek 2008, p. 361).

Since Williams' original proposals—Williams (1966) and Williams (1997)—Trope theory was employed to deliver a one-category ontology. Following Campbell (1990, pp. 20–21), tropes were designed as (i) *simple*, i.e., unstructured and partless;[7] (ii) *basic*, namely as the fundamental building blocks on which all the ontological burden of more complex (derivative) entities stands, such as concrete objects; (iii) *disjoint*, meaning that tropes are mutually independent or isolated items; (iv) *compresent*, for tropes of different types or kinds, when combined in groups or bundles in a concrete object are allowed to share their space-time location—see Denkel (1997) for an exhaustive account.[8]

So, in agreement with commonsense evidence, TO-ists hold that concrete objects like *this cat* or *that rabbit* are complex featured entities in that they are built up by the singular occurrence of certain basic items, i.e., tropes, singular cases of qualities and quantities. Concrete objects, in this picture, are "complex derivative entities", "total group[s] of compresent tropes" (Campbell 1990, p. 21), namely bundles (classes or collections) of tropes (*Bundle theory*). Each trope in a bundle "exists as individual entities at unique place-time" (Campbell 1990, p. 3) in the sense that

Footnote 5 (continued)
trast required a solution, and tropes were introduced to be a (the) presumed solution, for their peculiar nature (see Campbell (1990); Kuhlmann (2002) and Seibt (2002); Winter (2017) for appreciation and criticisms).

[6] In this respect, tropes seem to agree with empirical theories of force fields for they put concrete objects, like particles, as derivative from more vanishing and fundamental entities like fields.

[7] A substructure has formal but not (necessarily) ontological independence from the structure it constitutes, while a part is supposed to have ontological but not (necessarily) formal independence from the whole to which it belongs.

[8] Because TO metaphysics postulate only tropes as fundamental ontological category, it is not completely clear what is the status of space and time in such a theory. Is space-time a trope? Campbell seems to move in this direction interpreting space-time as field and then as tropes, i.e., sparse tropes that embrace the whole universe (Campbell 1990, p. 146). But, further questions then arise: is space and time also composed of tropes? If it is so, then, is there any privileged role that is played by spatial and temporal tropes in constituting concrete individuals? However, the present work is not pursued to answer such questions.

"they inhere in one [concrete] object only" (Rojek 2008, p. 364), but they can share their space-time location with other tropes with which they are "compresent". This peculiar behaviour is due to their specific particular but non-concrete nature. On the contrary, concrete objects or things have to satisfy some impenetrability principle, due to their material constitution, in terms of some principle of spatial exclusion: concrete objects saturate or "monopolize their location", being defined as "the totality of being found" (Campbell 1990, p. 3).

Yet TO's concrete objects does not support the metaphysical conception of concrete individuality endorsed by CN-ists and encoded by the principle of the *identity of indiscernibles* (PII).[9] According to PII, two entities sharing all the same properties (indiscernibility) are, actually, one and the same (identity), by some metaphysical necessity.[10] Campbell provides an intuitive trope driven counter-example to PII. Take two concrete objects, e.g., two diamonds, and recall now that within TO any concrete object is defined in terms of a bundle (collection) of *compresent* tropes, i.e., hardness, transparency, brilliance....mass, solidity...and so on—Campbell (1990) and Denkel (1997). Then,

> Because the items in the bundles are particulars, there is no problem with the Identity of Indiscernibles. A second diamond is a bundle of quite different particulars that resembles, more or less closely, the particulars in the first bundle diamond. Even if we adopt a thoroughly relational analysis of place in space-time, exactly similar diamonds with exactly similar space-time relations to exactly similar other tropes, will not be identical. So far from having all their bundles memberships in common, [...] these bundles are completely disjoint. Campbell (1990, pp. 20–21)[11]

Because of the peculiar nature of tropes, concrete indiscernible objects are numerically different by ontological necessity. As Armstrong notices: "Our selected simple trope, call it *a*, is numerically different from the others, but this numerical difference varies independently from its similarity or dissimilarity to these other tropes in the class" (Armstrong 2005, p. 310). So, because of their derivative nature, concrete objects inherit the inner numerical difference among tropes of the 'same' type.

So, in order to specify how tropes and derivative objects (formally) interact, TO must employ some other principle, intuitively weaker than PII. Furthermore, the explicit assumption of such a principle is an ontological (and formal) necessity, for it will allow us to specify TO's specific ontological space, by specifying *what* is

[9] But the converse holds for obvious reasons: it may be interpreted as a principle of 'nominal' identity saying just that if two different names refer to the same item, then the name distinction is inessential to distinguish that item and its characteristic role—as a property—in the constitution of concrete objects.

[10] See Wörner (2021) for a detailed account; Seibt (2002) provides a discussion of various ontological interpretation of PII, while Arenhart et al. (2019) provides a discussion about its unsuccessful application in fundamental physics.

[11] Campbell's concern with the relational space-time relations is an evidence of the problematic status of space-time in TOs. If space-time is a trope, then space-time trajectories are torpes too and, as such it may satisfy the failure of PII with opportune constraint on compresent tropes but, then, relations have to be considered as tropes too, and this generates the problems that I will discuss in Sect. 5; if space-time is not a trope, then what is the metaphysical status of space-time? Is TO complete?

Table 1 The ontological square

Entities	*Abstract*	*Concrete*
Universal	Formal Substances	Essences
Particular	Tropes	Individuals

the nature of TO's particulars, and how they differ from CN's concrete individuals. Accordingly, this principle must be based on a sufficiently strong relation to generate equivalence classes of indiscernible but still distinct particulars. Therefore, such relation must be weaker than identity.

Furthermore, a concrete object is predicated of a type (sort or kind) through the possession of certain (relevant) tropes. Namely, a concrete object is constituted of several (compresent) tropes, each of which, in turn, belongs to a specific equivalence class that allows for the predication in language.[12] This is, the issue of the nature of tropes is strictly related to the issue of predication, whose first specification I will consider in the next subsection.

2.1 TO's Predication: A First Sketch

A way to make clear that, among Trope theories, TO is particularly sensitive to the problem of predication is to pay attention to the idea that universals are usually indented to be the ground of predication; nonetheless, TOs do not flank tropes with further universal entities. Tropes are particulars and not universals and, as such, they ground the subject of predication, i.e., concrete objects. Consequently, tropes cannot play the predicative role directly. This can be regimented along the following line:

- $To \iff Tt \wedge t \in o$[13]

which says that a concrete object o is T if, and only if, the trope t of kind T is one among the constituents of o. Notice that T is the relevant predicate specifying the trope-universal i.e, a class of equivalence of tropes. Therefore, tropes themselves play a peculiar predicative role as the ground of the relevant predicate, although they are particular entities that enter in the constitution of the logical subject. This is

[12] Along these lines proceeds also the so-called *nuclear theory* of tropes (Kuhlmann 2002), which distinguishes among a class of essential tropes from the class of accidental ones, providing a metaphysical ground for the predication in TO together with an interesting account of change and dynamism (see Winter (2017) for a critical discussion).

[13] Tt is denotes $t \in [t] \iff Tt$ namely, that t belongs to the class of equivalence $[t]$.

even far more relevant in nominalist Trope theories like TO where equivalence relation generates the predicate T but, this time, although primitive they cannot play an autonomous predicative role (Sect. 4 is devoted to demonstrating such impossibility from the formal standpoint).[14]

According to TO, the appeal to classes—whether equivalence classes (universals) or bundles (concrete objects)—cannot have ontological or existential import and this imposes the discussion about how dismiss them from the ontological horizon. One very reasonable way is the following:

> Supporters of this view advanced a very reasonable way of reducing universals to tropes. [...] For trope theoreticians, appealing to classes has no ontological import. Talking about sets of exactly resembling tropes does not commit one to accept an extended set-theoretical ontology. This is only a way of speaking, which can always be replaced by speaking about similar tropes only. (Rojek 2008, p. 368)

This reductionist attitude to universals is synthesized by the *One Category Principle* (Campbell 1990, p. xi) according to which tropes constitute the "fundamental and irreducible category", "[f]rom which all the rest can be built or otherwise derived". Indeed, the nature of universals must be essentially derivative, they can be left as belonging to the level of language only.

Yet, differently from CN, and because of the twofold role of tropes, one as the ground for universals and another as of concrete objects, TO must provide some alternative nominalist strategy to interpret the universal 'talk'. Indeed, according to CN universals may be simulated by means of the *Leibniz Law* (**LL**):

$$(\mathbf{LL})\ x = y \longleftrightarrow_{df} \forall_F(F(x) \longleftrightarrow F(y)),$$

where first-order variables range over individual particulars while second-order variables range over higher-order entities like universals. Despite that **LL** is here expressed in a second-order language—given by the second-order quantification $\forall_F$ binding the second-order or predicative variable F—it is possible to reformulate it

[14] Tropes may cohabit with other sorts of entities, like natural classes Ehring (2011) the trope counterpart of realist universals. According to this approach, natural classes are entities over and above tropes, with the consequences of sensitively diverging from TO in general. For this reason, I won't discuss such an approach in full extension. However, let me motivate briefly the exclusion. Ehring (2011) holds that the nature of a trope is something external to it in the sense that it is determined by the membership of a trope to a natural class which, in turn, is taken as primitively distinct from (external to) the trope: the nature of a trope is built up as the set of all the natural classes of a trope. The following passage gives the insight: "The nature of a trope is identical to the natural classes it is a member of. So, the nature of trope t is determined by its class membership, but the trope's nature is not a constituent of the trope. The nature of a trope is just the set of natural types it falls under, but those types are not constituents of the trope" (Ehring 2011, p. 189). It is hard to see how to interpret natural classes and sets of them in a non-realist way, after all the set-theoretic membership relation is an external relation. For this reason, this peculiar variant of TO is not taken here as belonging to TO. Other examples of compatible universal entities are determinable universals (aspects) and concrete universals (essences). See, again, Rojek (2008) for appreciation.

for nominalist purposes into first-order terms, by employing first-order schematic formulas ϕ in place of quantified predicative variables.[15] In particular, the first-order reading of **LL**, provides a linguistic account of universals that is *prima facie*[16] consistent with CN's purposes in that it allows to characterize *what* concrete individuals are in terms of classes of "propositional" congruence, where universals are simulated by congruent classes of open first-order formulas. Notwithstanding this, the formal behaviour of equivalence classes of tropes cannot depend on the notions of individuality endowed **LL** because, despite that tropes t_1, t_2 may be of the same type T, i.e., $\forall_T(t_1 \in T \longleftrightarrow t_2 \in T)$ (indiscernibility), it does not makes t_1 and t_2 one and the same trope (identical).

However, TO employs an *equivalence class theory of universals* that is very close to that. For example, STO employs the *resemblance* class theory of universals according to the idea that (exact) resemblance a sufficiently appropriate equivalence relation to create the "illusion of (genuinely) common features" (Campbell 1990, p. 32) and, so, the "illusion of identity" (p. 43). Something analogous can be said about ITO, even though the (exact) similarity relation is now defined in terms of the more primitive equivalence relation of 1 : 1 proportion among quantity tropes:[17] ITO "replaces talk about 'exactly resembling' quantity tropes with talk about tropes connected by the relation of 1 : 1 proportion" (Keinänen et al. 2018, p. 578). Despite, then, that ITO is stronger than STO, ITO relies on the same assumption of some primitive equivalence relation and, so, it shares with TO the problematic thesis that trope-universals are equivalence classes of tropes.

But, is Trope Nominalism formally consistent? Certainly, to claim that allegedly derivative entities (like universals and concrete objects) are non-committing the theory to entities over and above tropes is per sé insufficient, if left standing alone, as a ground for the whole theory of predication. Namely, it is not sufficient to claim that equivalence relations are internal relations in order to make them as non-committing. What TO-ists owes us is a story about how the formal resources and the semantic strategies one has to employ to block the commitment to the aforementioned further set-theoretical ontology are appropriately interacting with the relevant ontology and metaphysics.

The issue concerning TO predication is deepened in Sects. 4 (but also 5, and 6). First, however, let me introduce the strategy that CN-ists employ to turn universals just "a way of speaking". This will shed light on the technicalities that make the issue of predication so complex within TO and, unfortunately, so problematic, as I will show in Sect. 4.

[15] Whether first- or second-order, **LL** brings together the (respective first- second- order version of) the already mentioned *metaphysical* PII and it converse, the *logical* law of the *Indiscernibility of identicals* (IIP), i.e., that two identicals, $x = y$, are *logically* indiscernible or share all the characterising properties – formally: $\forall_F(F(x) \longleftrightarrow F(y))$.

[16] As I will argue later, this move does not save CN from problems.

[17] The meaning of the relation will be discussed later.

3 The Nominalist Account of Predication

It is a common belief that the central thesis of Nominalism is that there are no universals (see Ferrari (2022) for a more articulated formulation of the whole issue). However, in formal ontology, CN holds a weaker claim: universals exist but not as entities over and above (concrete) individuals; they do exist just as *linguistic* entities or mere expressions. According to this approach, nominalist predication is restricted to a *linguistic* issue. This explains why Nominalists, in general, do have to provide *some* way—linguistic!—to assign values to the predicates that occur in the well-formed formulas of their theory. In other words, they have to provide some alternative non-realist or non-objectual assignment of values for those expressions. Following Barcan-Marcus (1972) and (1978), predicative expressions fulfill a linguistic role only in case any realist or referential claim is appropriately avoided by the semantics: the predicative expressions admitted in the language of the nominalist theory cannot be interpreted as entities of some further (and even higher-order) domain.

This had led many supporters of Nominalism to think that the minimal language and logic to appropriately represent the logical and grammatical role of their predicative expressions had to be that of predicate calculus, restricted to the first-order language *plus* identity. Such a restriction to first-order languages allows to avoid the commitment to second-(higher-)order entities—usually encoded by quantification over predicative or second-(higher-)order variables—from the outset, because such a quantification is simply not allowed by the first-order formation rules.

However, notwithstanding the restriction to first-order languages, Nominalists must provide a way to (formally) represent the predicative role of *open* first-order formulas[18] (see also Haack (1978)) which can be seen as *implicit definiens*[19] for each new non-logical or theoretical predicate constant to be introduced in the first-order language (Cocchiarella 2007, p. 85), every time it is demanded by the nominalist theory. Thus, Nominalists in formal ontology need to provide some formal method for the logical introduction of each—and all—new non-logical predicate constants as implicitly defined by characteristic open first-order formulas.[20]

As Cocchiarella (2007, p. 85) emphasises, a method to "account for all nominalistically acceptable predicative expressions", "without introducing each new predicate constant", does exists and can be formally set by extending the language and logic of the nominalist theory to a second-order language and logic, but where the predicative expressions—i.e., quantified second-order variables—are now *appropriately* interpreted (see also Quine 1964, 1981; Barcan-Marcus 1972). 'Appropriately', here, means 'in order to avoid the reference to entities belonging to the second-order domain(s)'. The standard way to account appropriately for this sort of interpretation is to appeal to a peculiar semantics, one such that operates a substitution of quantified predicate variables with first-order open formulas, technically called "substituents" (see Dunn

[18] I mean here, formulas with no bound occurrence of first-order variables, i.e., $\psi(x_1, ..., x_n)$.

[19] 'Implicit' for the relative definitions are not declared and formally stipulated from the outset, but instead they are implicitly introduced by means of axioms.

[20] Each predicate so 'defined' will denote a different sub-set of the first-order domain.

and Belnap 1968; Barcan-Marcus 1978; Haack 1978): according to this semantics, no commitment beyond that of substituend first-order formulas ψ is in place.[21]

As expected, to make it formally possible to have a substitutional interpretation of second-order expressions, some specific syntactic constraints are required on the characteristic principle of second-order logic, the *Comprehension Principle* (**CP**): "precisely those imposed on the comprehension principle in standard 'predicative' second-order logic" (Cocchiarella 2007, p. 85). Standard or impredicative **CP** posits properties and relations in terms of totalities to which they belong. In more technical words, **CP** works by postulating the existence of a subset of the first-order domain as the referent of the second-order (or predicate) variable introduced by a specification of its extension through a second-order formulas ψ in which bound predicate variables are allowed without restrictions as many definiens. Because of the lack of restrictions over the bound predicate variables possibly occurring in the second-order formulas ψ, the definiendum may occurr in the definiens as a universally quantified second-order variable, generating a problematic "circularity".

Thus, the mentioned *predicative* restriction is precisely addressed to block closed (second-order) formulas containing bound predicate variables from being used as the implicit definiens of the newly introduced predicate. Accordingly, **CP** is to be restricted by adding the syntactic restriction (**!**) on the occurrence of bound predicate variables in ψ, as follows:

$$(\mathbf{CP})\mathbf{!}\ \exists_{F^n}, \forall_{x_1,...,x_n}(F^n(x_1, ..., x_n) \longleftrightarrow \psi),$$

where ψ is a formula in which (**!**) no predicate variable has a bound occurrence, (1) F^n does not occur free in ψ, and (2) $x_1, ..., x_n$ are pairwise distinct object variables (Cocchiarella 2007, p. 86).

Condition **!**, together with (1), restricts the range of ψ to open first-order formulas. Furthermore, **!** works as the necessary (pre)condition for accessing the nominalist interpretation, otherwise frustrated *ab initio*.

But CN is not only a general thesis about the meaning of universals and their linguistic role. Something more is needed to provide the formal specification CN requires as an unbiased ontology. After all, the CN-ist must specify *what* entities are the value of first-order variables, and this must be done by specifying the behaviour of first-order entities using some additional axiom. According to Cocchiarella, again, this is the thesis of *extensionality* (Cocchiarella 1989). Extensionality consists in "the thesis that, semantically, predicate expressions may make no finer distinction

[21] Elsewhere, Cocchiarella (1989, p. 258) provides the formal specification of such an interpretation Nominalists should adopt:

Definition 3.1 (Nominalist interpretation) Let M be a model defined in the usual way and D_M the relative support or domain. Then a nominalist interpretation is an interpretation where a *substitutional assignment* is defined as a function s_N with the set of first-order and second-order variables as domain and such that

(I) For every first-order variable x, $s_N(x) \in D_M$;
(II) For every positive integer n and every n-ary second-order variable F^n, $s_N(F^n) = \langle \psi; x_1, ..., x_n \rangle$, for some first-order formula ψ and distinct individual variables $x_1, ..., x_n$ occurring free in ψ.

As desired, $s_N(F^n)$ makes the ontological commitment of the quantifier prefix $\exists_{F^n}$ nothing more than a way of speaking, an appearance.

of content (in the sense of the individuals such expressions can be true of) than can be generated by co-extensive predicate expressions"[22] (Cocchiarella 1989, p. 256).

However, CN-ists must mirror the extensionality thesis by means of an additional specific (non-logical) axiom of their theory, because (i) ontologists must specify what are the basic objects without presupposing their features *qua* objects—as if they were provided by some a priori "logical" apparatus—and because (ii) the formalized ontology cannot rest on relevant presuppositions about how it is able to maintain the coherence of its own characteristics thesis. First-order PII, arguably, plays that former role[23] while **CP!**, for example, the latter. According to Seibt (2002), PII is a principle governing concrete individuality in a way that it embeds a specific *metaphysical* thesis: that of substance metaphysics. A first-order formulation of PII, where first-order schematic letters ψ take the place of quantified second-order variables, is the following:

$$\text{PII} \quad \forall_{x_1,...,x_n}, \forall_{y_1,...,y_n}((\psi(x_1, ..., x_n) \longleftrightarrow \psi(y_1, ..., y_n)) \rightarrow x_n = y_n).$$

It should be noticed that schematics first-order letters ψ are very close to (if not the same as) quantified second-order variables with open first-order formulas as substituent (Haack 1978, pp. 53–54, p. 79). So, this formulation is consistent with CN approach to predication. PII, then, governs the individuation of universals in terms of the identity of those first-order entities that are pairwise satisfying congruent (or equivalent) formulas denoting some subset of the reference (first-order) domain.

To sum up, CN can linguistically simulate the distinctive role of realist universals using three connected operations: (i) the substitutional interpretation applied exclusively to second-order quantified expressions that is allowed by (ii) the syntactic restriction **!** (predicative **CP**), together with (iii) the postulation of first-order PII.

Unlike CN, TO does not support PII, as already mentioned. This result constitutes the main prima facie difference between CN and TO as nominalist theories. In the next section, I will discuss TO predication along the lines provided so far. However, before going ahead, it may be helpful to unearth some issues involved in CN's approach, which can enlighten later discussions.

[22] Cocchiarella continues: "This is the thesis, in other words, that co-extensive predicate expressions are to be interchangeable *salva veritate* in any applied formal theory of predication suitable for nominalism. This means that nominalism is committed to an extensional logic, and in particular to a non-modal form of the thesis of anti-essentialism; specifically, the thesis that no nominalistic universal is necessarily true of some of the things of which it can be truly predicated without being necessarily true of all."

[23] Notice that this principle requires to hold that the classical theory of identity (CTI) holds. CTI is given by *reflexivity* of identity (RI) $\forall_x(x = x)$ and the *substitutivity axiom* (SA) $\forall_{x_1,...,x_n}, \forall_{y_1,...,y_m}(x_i = y_j \rightarrow (\psi(x_1, ..., x_n) \longleftrightarrow \psi(y_1, ..., y_m)))$, that together are sufficient to characterize the logical concept of *congruence*. So, this axiom provides in the theory information about the "logical" suppositions required by the theory itself as an ontology. It is worth stressing that SA is the extensionality thesis and the converse PII. Notice, that SA is formally independent from **CP**, for it is formally provable by induction on formulas (Cocchiarella 2007, p. 32) once we accept the first-order nature of identity.

3.1 Nominalism and the Issue of Identity

A distinctive role in the success of CN is played by the identity relation occurring in PII. Indeed, CN-ists feel entitled to employ the nominalist assignment because they assume the identity relation as a first-order relation. This also allows maintaining **CP!** and PII as independent axioms of CN. Nonetheless, it is worth realizing that this reading is based on an additional presupposition: that the identity relation is purely formal, namely ontologically innocuous. In other words, that identity preserves a constant, *fixed* interpretation—independently from any context of application.

However, identity per sé cannot receive a first-order characterization (axiomatization) (Arenhart et al. 2019). This result is related to a theorem about first-order languages equipped with infinitary models—the Löwenheim-Skolem theorem. Accordingly, there is a fundamental ambiguity in characterizing identity as a first-order predicate: there cannot be a proof that first-order identity has a unique model (a set). This is, we have a disproof of the fixed character of first-order identity. The idea is that the formal resources of our theoretical language are not sufficient for providing a fixed and context-independent interpretation of the identity concept. On the contrary, a second-order characterization of the concept, by means of a definition of the sort of **LL** (Sect. 2.1), provides such an invariance. Nonetheless, any second-order predicate is provably categorical and, so, the identity concept cannot be read as a distinctive or logical predicate for its receiving a fixed interpretation. For this reason, identity, it is usually presupposed as *affine* to logical constant, despite being on a par with all the other first-order constants that constitute the "ideology" of a theory—like the membership relation in set theory.

As a matter of fact, then, it seems CN-ists have at hand only two alternative strategies to maintain identity as fixed or invariant: one that aims for a *logical* account and a second that invokes an *epistemic* ground. The logical strategy, as previously mentioned, solicits a second-order characterization of the identity predicate akin to that of **LL**. But this strategy contrasts with the nominalist spirit of CN, for the very reason that the employment of second-order identity means that the identity predicate is on a par with the other theoretical predicates (and relations) and, as such, it must be introduced by means of **CP!**. But, now, this gives rise to a contradiction among the explicit second-order characterization of identity, given by something like $\forall_F(Fx \longleftrightarrow Fy)$, and the predicative restriction **!** that does not allows for it. In the next section I discuss a similar result occurring within TO.

On the other hand, the epistemic justification holds something like what follows: despite of the impossibility of a first-order characterization of the identity concept, a first-order conception of identity is sufficiently solid because it is completely reasonable to ground identity invariance on the *a priori* nature of the concept. This move ultimately rests on a specific conception of identity due to the axiomatic approach, and sometimes called the *Fregean approach* (Rodin 2007). The Fregean view relies on the *presupposition* that the identity concept is somehow a priorily established as context-independent. Indeed, according to Frege, the whole of logic and mathematics are *a priori* and *analytic*. As Rodin (2007) recalls us, in the *Afterword* to his own *Grundgesetze* (Voll. II, 1903) Frege declares that "Identity is a relation given in such a determinate way that it is *inconceivable* that various forms of it

should occur" (Frege 2016, p. 254, emphasis my own). According to this option, the identity relation is given to us as logical inasmuch it is invariant, and the sole *presupposition* of such invariance is sufficient to ground the formality or logicality of the concept.

Compared to the first option, the epistemic move has the advantage that it is not interested in a second-order characterization of identity in order to secure its fixed meaning. Yet, at the same time, it has a cost: that is equivalent to say that our common pre-theoretical conception of a concept is sufficient to state something on the nature and properties of that concept despite that this cannot be investigated within an appropriate theory. The risk for CN-ists seems to be that, in the lack of a robust justification, identity can no longer be safely considered as both a first-order *and* invariant predicate.[24]

I will deepen the discussion on the relation among invariance and logicality later in Sect. 6.1, where I discuss in full extent the conditions and consequences for those TO-ists, like ITO-ists, that employ peculiar invariant (equivalence) relations to have kinds of tropes without kinds.

4 Problems with TO's Predication Theory

TO-ists assume primitive equivalence relations to "build up" those formal entities playing the role of realist universals. However, they must do it in a way that frees them from any commitment to the equivalence classes of tropes that constitutes the (second-order) meaning of predicative expressions.

According to the framework sketched so far, TO-ists' main concern is that equivalence relations do not seem to have an intuitively plain and fixed (*a priori*) interpretation. This is particularly apparent in the case of (exact) resemblance: it is not clear to what extent something resembles but is not identical to something else. There are no 'intuitive' reasons that justify the supposition that the equivalence relation is invariant in all contexts. Yet, Campbell's intuition behind STO is that (exact) resemblance shall be as close as possible to identity.[25] On the other hand, TO-ists

[24] For example, nowadays the epistemic or Fregean view of identity is far from being plainly and widely accepted. Rather, that is felt as too 'easy' for much mathematics and mathematical information is hidden in it. For example, A. Rodin, in Rodin (2007) (but also Ferrari (2022)), questions identity and rejects that view in favour of another "dynamical", i.e., one based on special morphisms in category theoretic setting. Unfortunately, such a view induces a completely different ontology, one based on relational structures rather than on particular individuals (Ferrari 2021). Once endorsed the epistemic view, then, one has the burden of reasonably defend it and give the precise conditions for such an acceptance.

[25] There are criticisms about this point. Rojek (2008) argues, correctly, that thinking of resemblance as an equivalence relation makes it too close to identity—in next section his argument is reported in more detail. However, if resemblance were non-transitive, as someone suggested, no way to simulate kinds and universals would be available. Yet, there are criticisms also to the tight proximity of identity and resemblance, like that (exact) resemblance behaves in a counter-intuitive way for it "violates the nominalist insight that all properties are not only *particular*, but also *different*." (Rojek 2008, p. 369). This latter criticism poses the basis of Rojek's work, purported to answer the question "how to define the criteria of similarity?". Interesting, Rojek's work, along mine, argues for the insufficiency of TO's proposal in this respect, emphasising the need for an additional category that of *aspect*. Details in his paper.

want equivalence relations to be as close as possible to identity because they want to make equivalence relations to reach and share the pure formality and ontological innocence of that latter. However, in the absence of strong 'intuitions' about its logicality, this last requirement can be satisfied only through a second-order characterization of the primitive equivalence relation. But this, in turn, raises the question of how to make the second-order nature of equivalence relation(s) compatible with the nominalist austerity. These concerns are grounded on the problems that emerge from the application of the substitutional strategy discussed above to the case of TO predication theory.

The fact that TO-ists give a distinctive account of universals as equivalence classes forces also a reformulation of PII as the axiom that governs the specific behaviour of TO's first-order entities in terms of some binary predicate, say '$=_{ER}$', for equivalence relations, like (exact) resemblance (STO) or 1 : 1 proportion (ITO):

$$\text{PII}_{ER} \quad \forall_{x_1,\ldots,x_n}, \forall_{y_1,\ldots,y_m}((\psi(x_1,\ldots,x_n) \longleftrightarrow \psi(y_1,\ldots,y_m)) \rightarrow x_i =_{ER} y_j).$$

Following this line of thought, then, the non-logical relation $=_{ER}$ has to satisfy two formal requirements: (a) $=_{ER}$ has to share some, say, *minimal* formal property of identity, namely has to behave as an equivalence relation, and (b) $=_{ER}$ must be a purely formal relation, namely it must receive a fixed interpretation from the outset. It is worth emphasizing that condition (a) per sé, does not commit TO-ists to a second-order language and logic.[26] Condition (b), instead, raises our concern. That requirement forces TO towards the aforementioned logical strategy. This means to characterize $=_{ER}$ along the line of **LL**, where now variables x, y range over tropes (TO's first-order entities) and the second-order variable T over types of tropes—which allow for predication:

$$(\mathbf{ER}) \quad x =_{ER} y \longleftrightarrow_{df} \forall_T(T(x) \longleftrightarrow T(y)).$$

Now, $=_{ER}$ is a primitive (non-logical) predicate of TO and, so, according to the substitutional strategy, it has to be formally introduced by **CP!**. In this case, the application of the nominalist constraint **!**—the one that should prevent the objectual interpretation of the second-order quantifiers, now implicitly occurring also in PII_{ER}—is contrasted, though. Indeed, the ψ occurring in **CP!** has to be substituted by a formula like the bound second-order formula occurring as the definiens of $=_{ER}$ in **ER**. Indeed, by opportune substitutions, we obtain:

$$\mathbf{CP!}_{ER} \quad \exists_{=_{ER}}, \forall_{x,y}(x =_R y \longleftrightarrow \underbrace{\forall_T(T(x) \longleftrightarrow T(y))}_{\psi}),$$

where the (formal) *contradiction* is manifest. According to the predicative restriction (**!**), the formula ψ on the left-hand side of the main bi-conditional must be a

[26] It says, instead, that to make a given relation an *equivalence*, it is enough to introduce the first-order properties of reflexivity, transitivity and symmetry for that predicate, as additional first-order axioms of the theory.

first-order formula, one in which no predicate variable has a bound occurrence.[27] However, according to constraint (b) and the fact that the relation is not logical, ψ has to be substituted by formula that has bound occurrences of the second-order variable T.[28] Therefore, TO cannot appeal to the substitutional strategy. As a further consequence, TO cannot preserve the formal independence of **CP!** and PII_{ER} for obvious reasons.

As mentioned, PII_{ER} is a requisite of TO because it allows TO-ists to specify what are the basic entities of the theory. But, on the other hand, it is the acceptance of PII_{ER} that produces the problems.

Looking for an alternative solution to TO's account of predication, TO-ists disagree on how to solve the problem. On the one hand, STO-ists seem to prefer the way that gives up to the semantic strategy: they appeal to the supervenience claim. On the other hand, ITO-ists seem to pursue a solution that allows for keeping the semantic strategy, by giving up to all the problematic second-order talk in one go: they appeal to the invariance of the primitive equivalence relation along the line of the epistemic strategy. In the following sections, I will discuss both options.

5 The Appeal to Supervenience

STO assumes (exact) resemblance (ER) as its peculiar theoretical "*non-eliminative* (i.e., primitive)" (Campbell 1990, p. 38) equivalence relation. Equivalence relations are formal as long as they are invariant and, in turn, invariance can be granted only by their second-order nature. In the face of the acceptance of second-order entities STO-ists have the burden of proof that entities over and above (first-order) tropes are metaphysically innocuous, that second-order entities do not possess causal powers. Causal powers belongs only to tropes. In the contrary case, TO-ists were engaging a dualist metaphysics that would explicitly go against the naturalist inspiration from which it originated. A compelling argument for the metaphysical inefficacy of higher-order entities would allow to consider types of tropes as *pseudo-addition* to the metaphysically relevant first-order ontology and, so, it would help to read equivalence relations as internal. Such an interpretation seems to be compatible the following commentary about the *prima facie* derivative-primitive contrast in TO. As Maurin writes:

> In the beginning of this section on resemblance it was pointed out that the account rests on two presuppositions: that resemblance is objective[29] and primitive. But if we adopt the alternative according to which exact resemblance is nothing but a pseudo-addition to our basic metaphysical scheme, it may appear that we have abandoned the presupposition that exact resemblance is primitive. As Campbell notices, this alternative seems to accord resemblance both a derivative and a primitive status. Resemblance is derivative in the sense that

[27] The other conditions on ψ are satisfied for $=_{ER}$ does not occur free (1) and both x, y are free (2).

[28] It is easy to see that an analogous contradiction happens to CN, when substituting LL to **ER**.

[29] In the sense of being independent of the specific context of interpretation and, so, fixed.

> it is a 'pseudo-addition' to that which it relates. We might say that, as long as truth-makers are what interest us, the relation of exact resemblance is 'reducible' to that which it relates. But it is also primitive in the sense that no eliminative definition of resemblance is available. Treating exact resemblance as primitive in this latter sense means having to accept a host of formal characteristics as indefinable. (Maurin 2002, p. 115)

About such formal characteristics, Armstrong specifies that

> [T]hese features [i.e., the indefinable formal characteristics] of resemblance are part of the ontological cost of the theory. For these features have to be taken as primitive, and therefore unexplained axioms of resemblance. Particularised natures are such that they necessarily sustain resemblance relations obeying these axioms. A certain theoretical burden, therefore, is placed on the particularised natures. (Armstrong 1989, p. 57)

Campbell seems to be aware of all the problems so far discussed. He explicitly invokes the superveneience claim as a solution for the primitive-derivative nature of resemblance: (exact) resemblance supervenes on tropes (Campbell 1990, p. 38) and "[s]upervenient 'additions' to ontology are pseudo-additions" (Campbell 1990, p. 37).[30] Internality, supervenience and metaphysical innocence are supposedly connected. Armstrong, again, writes that "where we have an internal relation, [...] [t]he relation supervenes upon the terms [...]. That, I think, makes the relation an ontological free lunch" (Armstrong 1989, p. 100).

It is worth noticing that the context in which Campbell appeals to the supervenient status of (exact) resemblance is that to reply to the *Küng's argument*, an argument that poses a perspicuous problem to TO-ists.[31] The argument questions whether it is plausible to set a sharp boundary to exclude relations from receiving a genuine trope interpretation. In fact, the abstract nature of tropes would allow for the inclusion among tropes of relations and among these, of course, of (exact) resemblance, as a peculiar instance. However, once interpreted relations as tropes—Küng's argument proceeds—one shall obtain something like an infinitely augmenting (uneconomical) expansion of the trope domain from a very trivial case. Suppose that both properties and relations are tropes, then,

Küng's argument. Assume three tropes of the same type, i.e., shades of a specific colour, a, b, c. Then, given that *ER* applies to tropes, $ER(a, b)$, $ER(b, c)$, $ER(a, c)$ are obtained. However, if relations are tropes, then $ER(x, y)$ is a distinctive trope for any trope x, y. Thus $ER(a, b)$, $ER(b, c)$, $ER(a, c)$ are different tropes themselves of a certain type, i.e., of a specific colour. Therefore, given that *ER* applies to tropes, *ER*[

[30] It is widely known that the concept of supervenience is very often used in anti-emergentists and neo-positivist (neo-empiricist) inspired environments in order to reductively explain what is commonly considered to be an emergent and collective (higher-order) behaviour, over and above their constituents. Such behaviour, by using supervenience, is usually understood as metaphysically inert and so epiphenomenal.

[31] The argument is a reformulation of the famous Russell's resemblance regress to be found in Russell (1997, p. 48).

$ER(a, b)$, $ER(b, c)$], $ER[ER(b, c), ER(c, a)]$, $ER[ER(c, a), ER(a, b)]$ are obtained. But then, they are tropes too ... and so on *ad infinitum*.

An analogous argument appeared in (Hochberg 1992, p. 194), but this time the argument moves from two tropes.[32]

Hochberg's argument. Suppose both properties and relations are tropes. Then *ER* is a trope, t_i, that belongs to the class **ER** of (exactly) similar tropes $t_1, ..., t_n$, that are all numerically different cases of *ER*. Nevertheless the class **ER** is defined by the predicate '*ER*': $\mathbf{ER} =_{df} \{\langle t_i, t_j \rangle : ER(t_i, t_j)\}$ with $i, j \in \mathbb{N}$. Then, an infinite generation is automatically obtained: from any two such tropes, a third always exists, the one that relates the former two as resembling each other and that makes them belong to the class.[33]

Hochberg seems to reply to Armstrong's partial indifference to the ontological costs of TO, when he comments that "[t]he superfluous generation of infinitely many entities from one or two required entities is an embarrassment for any theory" (Hochberg 1992, p. 194).

However, the relevant point here is that a STO-ist should provide reasons for the in-principle exclusion of some or all relations *among* tropes *from* receiving a trope interpretation. Relying on the supervenient nature of equivalence classes (universals), Campbell's reply to Küng is that such an 'infinite generation' is "*no more burdensome*" because just "*automatic inevitable*" (Campbell 1990, p. 37), namely formally inevitable but metaphysically irrelevant. By taking (exact) resemblance as supervenient, the STO-ist seems to be able to block both the arguments from the outset, as long as supervenience does provide a criteria to decide which amongst properties and relations are to be interpreted as tropes and which cannot: supervenient properties and relations are internal and, as such, they are metaphysically irrelevant.

Nonetheless, the status of the supervenience claim, whether true or false, together with the adequacy of a suitable definition of supervenience are usually left unquestioned. Investigating whether the supervenience strategy is successful is the purpose of the next section.

5.1 Problems with STO

A recent discussion of the related concepts of emergence and supervenience is provided by Kim (2006), among others (see Campbell and Bickhard (2011) and Campbell (2015) for detailed full-fledged accounts). In a nutshell, Kim provides an argument that starts from the assumption that supervenient and irreducible higher-order properties are causally efficacious and concludes that such an assumption is inconsistent with our best model of causation. More specifically, a case of systematic causal over-determination is obtained. Causal over-determination contradicts one of the principles governing causality, the *causal exclusion* principle, that states that there

[32] Contrary to my work, Hochberg's does "not propose to discuss those versions of 'trope theory' that seek to avoid relational properties" (Hochberg 1992, p. 193) and is not focused on the theory of predication.

[33] Notice that, by diagonalisation, the defining predicate never belong to the class.

cannot be any metaphysically relevant case in which *two independent* causes produce *one* and the same effect. The basic point of Kim's argument is that causal exclusion prevents causal over-determination because that principle is *analytic*, namely a necessary and *a priori* truth. Therefore, the causal efficacy of supervenient properties must be abandoned.

However, many different analyses of the argument proves that the argument is invalid (Corry 2013; Horgan 1997) and unsound (Campbell and Bickhard 2011; Corry 2013) and, so, that it cannot be used to argue for the metaphysical inefficacy of higher-order entities. However, for our purposes it suffice to focus on the related metaphysical adequacy of the notion of supervenience.

Kim is deeply skeptical about the suitability of supervenience. According to Kim himself, supervenience is a "negative condition" (Kim 2006, p. 557) of higher-order causality because the claim "that a family of properties supervenes on another does not tell us much" (Kim 2006, p. 556). Understood as a dependence relation, even of higher-order properties like universals on first-order particulars (tropes), supervenience encapsulates so many different—and, so, non-homogeneous—sort of dependence relations that "the only thing common" to such variety is the fact that supervenient properties "covary in a certain way with naturalistic properties" (Kim 2006, p. 557), namely our tropes as long as they are first-order properties Kim (2006. In other words, the supervenience claim cannot go further than claiming that if two particulars are indiscernible with respect to some metaphysical relevant property, then they are indiscernible with respect to the supervenient property. So, Kim complains "that there is an in-principle unexplainable covariation between the putatively emergent [higher-order] properties and their base properties" (p. 556).

Clearly, in the context of TO, the role of basal particulars and their basal properties is played at once by tropes, while the role of supervenient (or emergent) properties is to be intended, for the sake of argument, as played by the universals generated by the primitive equivalence relation.

However, Kim is right in being suspicious. As Bickhard (2021, p. 130), points out, this simple asymmetric co-variation is "too weak to capture the intuition of some dependence: any lawful dependence would satisfy it". That is, supervenience fails to characterize any "deep" metaphysical or ontological relation of dependence: as a covariation relation, supervenience can at most capture some functional input-output co-variation, so that it "fails to capture the sense in which there is supposed to be an ontological dependence". The consequence is that "even when there might be such a dependence" the supervenience claim just states such dependence but without explaining it. Supervenience, then, is a black box unable to explicate any ontological dependence at work. In the absence of an appropriate explanatory "mechanism", the supervenience claim is simply metaphysically uninformative.

Furthermore, the appeal to the supervenience claim presupposes the definition of supervenience to be metaphysically adequate, indipendently from any specific characterization of the supervenient base. For example, when we have to discern between those relations that are external and those which are internal. But, given that the definition of supervenience is consistent with the particularist and materialist assumptions, according to which "a purely physical system" is "composed

exclusively of bits of matter" Kim (2006, p. 548) and, more in general, of particulars, then the supervenience relation cannot be called to play such a delicate role. Indeed, relations, whether internal or external, are collective and not particular properties. So, the notion cannot be applied.

However, one may argue, understanding supervenience as a co-variation relation does suffice to distinguish among internal and external relation, according to the following argument. Take the longest pencil in a given box. "To be the longest (in the box)" is clearly a relational property of a particular pencil among others, if any. Nonetheless, co-variance is falsified when an even longer pencil is introduced in the same box. In fact, despite the supervenient base of the former longest pencil does not change, the relational property clearly had: that one is no longer the longest pencil (in the box). So, "to be the longest (in the box)" is clearly an external relation. Therefore, external relation are not supervenient and supervenient relations are internal.

This does not suffice to make supervenience to hold, though. Indeed, there is plenty room for relational properties that does not fit with the definition of the supervenience base. For example, it is hard to argue that my own existence as myself does not strongly depend on the surrounding external environment(s)—whether cultural, social, biological, physical, and so on. Vice versa, how much do I essentially contribute to (each of) these environments? Is the relation between me and my environment internal? After all, it necessarily depends on my-self even if conceived as a bundle of particulars, as well as on all the particulars that constitute the specific surroundings (and vice versa). But, still, the relation itself is causally efficacious. The current best theory of biological foundations, labelled as *Extented Synthesis* (see for a discussion, Winter 2017), provides solid naturalist basis for this interactive or relational view. Thus, what relation does it count as internal, and what as external? Where is and how can be set up that boundary? And, in case they are not internal, what blocks the possibility that many of these relations cannot be captured by co-variant relations?

Supervenience cannot discern among metaphysically relevant and metaphysically irrelevant relations. So, the appeal to supervenience has little to do with picking up internal and metaphysically inert relations, like basic primitive equivalence relations. Accordingly, then, the appeal to supervenience is ineffective and STO begs the question of a semantic strategy to unload TO from higher-order commitments.

5.2 A Further Drawback of (Exact) Resemblance

The problems of STO with respect to (exact) resemblance may force TO-ists to get rid of the fixed meaning of the primitive relation. However, in this case a further concern emerges. The idea is that any formal specification of the equivalence classes generated by the resemblance relation is wholly determined by the sole peculiarity of the tropes belonging to the class. Supporters of TO, however, seems to be unjustified in doing so. As Rojek observes certain oddities can be derived from this view. Rojek writes:

> Two tropes of determined red resemble exactly one another. What about the trope of a determined shade of red and the trope of a determined shade of green? They belong to one universal, namely color, made with the help of exact resemblance, so they should also resemble exactly one another. But, first, it is strange to claim that a relation of the same kind (though of different degrees) holds between two red tropes [red-resemblance] and between a red trope and a green one [colour-resemblance]. It may be thought that exact resemblance holds, if any, only for two tropes of the same shade of red. (Rojek 2008, p. 369)

Notice that such drawback does not depend on the peculiar nature of tropes but, instead, it depends on the lack of a invariant interpretation of (exact) resemblance.[34] In this case, the behaviour of (exact) resemblance is strongly unstable and it is hard to see how the presumed distinctions among types of tropes do not come to collapse. The argument may be semi-formalized as follows.

Collapse. Be tropes a_1, a_2 of type A and tropes b_1, b_2 of a type B and in general be $A = [x]_{ER} =_{df} \{a_i : ER(x, a_i)\}$ and $B = [y]_{ER} =_{df} \{b_j : ER(y, b_j)\}$, with $i, j \in \mathbb{N}$. That is, that A, B are the equivalence classes, e.g., $[x]_{ER}$ and $[y]_{ER}$, of tropes made by ER. Recall that ER applies to tropes only, e.g., $ER(a_1, a_2)$ and $ER(b_1, b_2)$ are well-formed but $ER(A, B)$ is not. In addition, suppose α to be the *type of types* A, B.α, as a type (universal), has to be defined using ER as follows: $\alpha =_{df} [w]_{ER} =_{df} \{a_i, b_j : ER(w, a_i) \vee ER(w, b_j)\}$. Tropes $a_1, ..., a_n, b_1, ..., b_n$ are all elements of α. But then, it is not clear how to maintain that $A \neq B$ and, even, $A \neq B \neq \alpha$, for all those types are equivalence classes made with the help of the same equivalence relation ER and, hence, all tropes in α must be mutually similar, e.g., $ER(a_i, b_j)$.

The distinctions among same or cross-level types of tropes do collapse once the specification of the *type of types* is made dependent exclusively on tropes. Perhaps, as Rojek comments, the TO-ist "requires many kinds of exact resemblances and must take them as primitive". Two issues may be noted here. First, introducing many different (exact) resemblance relations will break the formal neutrality of ER, by fragmenting it in a variety of peculiar cases. Second, in this way similarities and distinctions no longer will depend exclusively on tropes.[35]

Now, we have all the reasons to look for an alternative approach to TO's predication, one which is also equipped with an alternative equivalence relation.

[34] The difference between this and Hochberg's argument is that in the former the resemblance does not receive a trope interpretation as it gets in the latter.

[35] However, this solution implies, according to the axiomatic method, that the formal theory shall be equipped with as many predicates as primitive relations and, then, as many axioms governing their behaviour as equivalence classes.

6 The Appeal to Invariant Equivalence Relations

Unlike STO, ITO moves from the idea that primitive equivalence relations are internal because invariant despite their being first-order. Two recent works on the topic by Keinänen et al. (2018) and Keinänen et al. (2019) are prominent in this trend. Unlike STO, such a distinctive approach to TO universals is based exclusively on "two different kind of necessary relations" (Keinänen et al. 2018, p. 577), those of 1 : 1 proportion and order between quantitative property tropes. Unlike STO, ITO takes such relations to be internal but not supervenient. According to ITO, both relations suffice "in characterizing the 'similarities' between quantity tropes and their belonging to determinate/determinable kinds" Keinänen et al. (2019, p. 530). Moreover, by their interplay it is possible to unify and distinguish (by proportion) and organize (by order) trope reality (Keinänen et al. 2018, p. 580). Accordingly, because ITO gets rid of both (exact) resemblance and supervenience, ITO does not seem to encounter the problems previously discussed. For this reason, Keinänen, Hakkarainen and Keskinen feel very confident that ITO is much superior than STO:

> Our account presented in the previous section provides the best currently available conception of "similarities" between quantity tropes. With the help of basic internal relations of proportion and order, we obtain an accurate conception of similarities and eliminate any reference to determinables. Hence, we explicitly eliminate any need for assuming determinable universals. (Keinänen et al. 2019, p. 533).

ITO is not committed with kinds because, argaubly, the primitive relations are internal and, "[t]herefore the relations of proportion and the relation of order remain invariant irrespective of the conventionally chosen [quantity] unit" (Keinänen et al. 2019, p. 527) (see also Keinänen et al. 2018, p. 577). "Invariance" here must be taken in the precise sense of "uniquely specify[ing] the determinate/determinable quantity under which given tropes fall" (Keinänen et al. 2019, p. 531). This is to say that proportion and order are primitive and they do have a fixed, context-independent interpretation, although they are of first-order nature. That said, the invariance is a desideratum of ITO because it also helps to avoid the suspicion of circularity in characterizing the similarities (Keinänen et al. 2019, pp. 531), as either exact or inexact (as in the case of quantitative distance), because it leaves (exact) resemblance with no role "(even considered as a derived notion)" (Keinänen et al. 2018, p. 578).

This approach is innovative but, I think, few more words about the basic relations of proportion[36] among quantity tropes are needed in order to understand whether such a circularity is avoided in the foundations.

Let me provide some further specification of the proportion relation. According to ITO-ists, 1 : 1 proportion is an equivalence relation that "can be expressed by means of real numbers" (Keinänen et al. 2018, p. 477). So, in addition to the specific axioms governing the first-order quevalence relations, ITO implicitly assumes all the mathematical theory of reals.[37] A first concern might be that to think of such a theory as one being implicitly committed with second-order entities since 1 : 1 proportion might be taken as a distinctive "second-degree relation universal" (Keinänen et al. 2018, p. 577 fn. 9). In this case, a problem for the nominalist inspiration of ITO would emerge because, despite that the second-order nature secures the invariance of proportion relation, equivalence classes of tropes are to be interpreted as distinctive entities in addition to tropes. However, the Authors claim this is not the case because the proportion relation is internal (Keinänen et al. 2018, p. 577, fn. 9) and, then, invariant. But, then, the following dilemma arises: if a non-logical equivalence relation is taken to be invariant, then either it shall be a second-order relation and, so, as belonging to a distinctive reality—with the immediate consequence of being an external and not an internal relation—or it is of first-order nature but, then, the relation cannot be safely considered as invariant: a compelling argument for the invariant character of the first-order equivalence relation must be provided. In both cases, ITO has a problem. In the first case, ITO-ists must argue for the metaphysical neutrality of the relation, because of the aforeseen failure of the substitutional strategy meant for discharging the ontological commitment over and above that of tropes. But, in this case, it is hard to figure out how the ITO-ist can argue for that and avoid problems analogous to those encountered by STO-ists. In the second case, ITO-ists might undertake a path analogous to the epistemic strategy of Fregean inspiration, consisting in presupposing the invariant nature of the first-order equivalence relation. Nonetheless, in this case the ITO-ist needs to provide evidences that

[36] In what follows I will restrict the discussion to the relation of proportion. One of the reason is that the order relation cannot works without the former. It is in some sense dependent on the characterization of the trope reality as provided by the proportion relation. A second reason is that the relation of order assumed here is defined as *equal to or greater than* (Keinänen et al. 2018, p. 577), namely it is a strict-order. Consequently, together with the usual axioms governing the strict order relation, ITO is equipped with the identity relation together with its distinctive "logical" axioms of reflexivity (RA) and substitution (SA) (Arenhart et al. 2019). But the status of the identity relation is of major interest and concern for Nominalsts as argued in Sect. 3. That discussion cannot be dismissed.

[37] A. Tarski proved that the theory of reals (R) can be a complete theory (in the sense of Gödel). Nevertheless, as Smith comments, this means that there cannot be a predicate of R which picks out the natural numbers: "while the real numbers contain the natural numbers, the pure theory of real numbers doesn't contain the theory of natural numbers" (Smith 2007, pp. 157–158). Namely, in the case one might think that ITO is assuming Tarski's complete axiomatization of the reals, then ITO could not speak of countable units and, so, of the relation of 1 : 1 proportion among countable and discrete quantities is ill-defined. In the alternative case in which R may speak of countable units, then R cannot be a complete theory, because of Gödel's first incompleteness theorems, but it still relies on the presumption that the identity relation is invariant. As I will discuss the next subsection, this presupposition also constitutes a problem for ITO-ists. Therefore, in both cases ITO has a problem.

the source of invariance is independent from the intended ITO's ontology, on pain of begging the question.

The discussion of a general criterion of invariance for first-order equivalence relations together with a discussion of its suitability in this framework is presented in the next subsection.

6.1 Problems with ITO

In order to assess the availability of ITO, let us take a step back to what I called the epistemic move (Sect. 3.1). According to this move, we are free to assume primitive equivalence relations for universals as first-order relations only under the presupposition that these are logical. The case of identity (Sects. 3–3.1) was enlightening, considered that it is a peculiar equivalence relation. However, things are different with respect to generic equivalence relations: it is not so safe to argue for their logicality and the case of *Collapse* is only one among the symptoms due to such impossibility. Nonetheless, equivalence relations, if provided that they are invariant, can be considered ontologically innocuous (*a priori*) and context-independent (formal).

For example, with the aim of providing a ontologically independent argument for the invariance of logical constants, logicians and philosophers have devoted many efforts since Tarski with his famous 1966 article *What are logical notions?* (Tarski 1986). Tarski inaugurated an entire cottage industry of Fregean inspiration, that has been more recently continued and extended by Sher (1991) and (2021). Tarski and Sher marshalled the meta-logical thesis that relations are logical if characterized in terms of a specific sort of invariance. In particular, they formulate a semantic "criterion" of logicality. Tarski writes: "I suggest that [...] we call a notion 'logical' if it is invariant under all possible one-to-one transformations of the world onto itself" (Tarski 1986, p. 149). More specifically, "[t]he invariance used in this thesis is, essentially, invariance of properties under $1-1$ and onto replacements *of individuals*" (Sher 2021, p. 1, emphasis my own). Clearly, Tarski's account of logicality presupposes a specific notion of particular individual and, as such, it might well suit ITO-relations. In fact, as noticed by Dutilh Novaes, Tarski suggests there that the resulting logic "is essentially about *quantities*, about *numbers*" (Dutilh Novaes 2014, p. 84).

The point now is to understand whether the Tarskian criterion provides ground for invariance that is ontologically independent from and, so, not restricted to that of ITO (and CN). Unfortunately, as I will argue, the answer is negative.

As Dutilh Novaes points out, Tarski himself seems to be aware of some problem about the generality of his approach when, in that article, he questions the conceptual adequacy of the permutation invariance criterion. Tarski writes: "it turns out that the only properties of classes (of individuals) which are logical are properties concerning the number of elements in these classes" (Tarski 1986, p. 151). It is precisely here that we see how permutation invariance works as a criterion of logicality. Namely, the only properties or relations the criterion 'makes' logical are those that apply to a specific class of entities: "all that matters with respect to an object is its

'one-ness', i.e., what is traditionally known as its numerical identity: the fact that it is one individual object." (Dutilh Novaes 2014, p. 85). In essence, as long as the criterion is "only sensitive to the number of elements in a class of individuals (and thus can only differentiate classes of individuals of different sizes)" (Dutilh Novaes 2014, p. 85) its validity is restricted to classes of well-defined particulars.[38] That is, the criterion presupposes specific characteristics of those entities:

> These [features] are: objects must be discrete; they must perdure; they must not merge (two objects becoming one); they must not multiply spontaneously (one object becoming two). As obvious as they may seem, these are substantive metaphysical assumptions on the nature of objects, which are clearly presupposed by the criterion of permutation invariance; essentially, the number of objects in a class must remain stable, otherwise the criterion simply breaks down. Not surprisingly, these are precisely the features of countable objects. [...] [T]he criterion is not ontologically neutral. (Dutilh Novaes 2014, pp. 85–86)

Unsurprisingly, these are precisely the features that ITO (and CN in general) attributes to its own basic entities: quantity tropes are simple and disjoint particulars and, so, are countable quantities—indeed they are stable, constant. So, as well as for the identity relation, the 1 : 1 proportion might be considered logical (in the Tarskian precise sense of 'invariant'), context-independent, and ontologically innocuous *only if* the relevant characterization of the basic entities of ITO as particulars is *presupposed*.

Said that, let us draw some further consequence. Recall the ITO-ists hold that "[o]ur main thesis is that as determinate particular natures, quantity tropes falling under a single determinable (e.g. rest mass) are all mutually connected by" the relations of proportion (and order) (Keinänen et al. 2018, p. 577). Recall also that ITO aims to "replaces talk about 'exactly resembling' quantity tropes with talk about tropes connected by the relation of 1 : 1 proportion" (Keinänen et al. 2018, p. 578). Consider now the thesis discussed above that the relation of 1 : 1 proportion (and order) is invariant under the permutation criterion and that the criterion, in turn, presupposes a certain notion of quantity. Then, it becomes clear that the relation of 1 : 1 proportion cannot contribute by any reliable means to characterize quantity tropes, because it presupooses them to be kept first-order and invariant. The very question is, then, the following: what makes quantity tropes the entities ITO is about?

The fundamental crux of ITO's theory of predication is, then, the following. Quantity tropes are those tropes that, depending on their quantity, cannot fall under more than a single *kind* of quantity because the quantity unit must be fixed in advanced in some way. After all, the 1 : 1 proportion supposes the fixed notion of *unitary quantity*, as "irrespective of the conventionally chosen [quantity] unit" (Keinänen et al. 2019, p. 527). However, to fix the quantity unit is to provide some fixed "one-ness"—independently from any conventional choice. But, then, some

[38] "Well-definite" in the sense of Keranen (2001).

axiom like PII_{ER} or PII are required for governing the numerical identity of the basic quantity unit. Nonetheless, this notion must be invariant, and so, independent from the linguistic resources at hand. Therefore, in order to fix the quantity unit, ITO is committed back to an equivalence relation of the sort of **ER**, namely one definable in second order terms.

In conclusion, I can say that there are reasons to affirm that first-order invariance of equivalence relations like 1 : 1 proportion (and order) presupposes some more primitive notion of similarity that allows to fix, once and for all and from the outset, the fundamental notion of unitary quantity that is at the basis of ITO. And this will take back ITO to share the previously discussed problems of STO or even of TO. Unfortunately, in this way even a more general contradiction arises within ITO: the allegedly avoided circularity between (exact) resemblance and 1 : 1 proportion comes back here in a new fashion.

One last comment. The ontologist may wonder whether it might be of some help to the nominalist project if quantity units were provided by some mathematical apparatus to be flanked to ITO—or to any other nominalist ontology. One simple reply: that would not help the nominalist project. In that case, indeed, the mathematical apparatus would count as an essential part of the (formal) ontology.

7 Conclusion

The discussion engaged so far, if correct and sound, unearths some crucial problems with TO's theory of predication, whether in the general form (in the first place) or in both the more specific trends of STO and ITO (in the second place). Consequently, the formal coherence of one-category trope ontology seems to be affected, perhaps, irremediably.

The first part of the paper (Sects. 2–4) was devoted to providing the essentials of TO, with a particular emphasis on its predication theory and related problems. In particular, in Sect. 3, I presented the general account of predication that fits the nominalist purpose of discharging higher-order commitments, together with a discussion of the presuppositions about identity relation. In Sect. 4, I come back to TO and presented an first infelicitous result for TO-ists, and according to which the interplay between TO predication and the nominalist strategy brings a contradiction that, in turn, arguably is what provides solid basis for the further development of TO into the two main branches of STO and ITO. From thereafter, then, the paper has been split into two comparable but (partially) autonomous sections.

On the one hand, Sect. 5 discussed STO, which historically comes first: according to STO-ists that very contradiction and further related arguments may be made innocuous by appealing to the supervenience claim: universals supervenes on basic tropes and, so, are not metaphysically committing the theory with ontological realms over and above the first-order one of tropes. If supervenience works, then higher-order entities, despite being unavoidable, may be considered as metaphysically inert and, so, irrelevant. Nevertheless, in Sect. 5.1, I show that things are not so easy because supervenience does not explain so much and, worst, it cannot discern among metaphysically relevant and metaphysically irrelevant relations. So, supervenience is

of no help in order to withdraw trope-universals form the metaphysical scheme. As a consequence, STO begs the question of a minimal semantic strategy of the kind discussed in Sect. 4 with all the problems it brings. Section 5.2 provided some further drawbacks of the specific equivalence relation of (exact) resemblance.

Doubts about STO, together with the problematic features of (exact) resemblance, can be thought of as what pushes for the development of ITO. So, Sect. 6 introduced ITO in order to expound the alleged advantage of assuming an alternative account of invariant primitive equivalence relations. Nonetheless, in Sect. 6.1, I have argued that ITO does not suffices in restoring the former problems of TO, because it lacks an ontologically innocuous ground for making the basic equivalence relation invariant. At most, ITO-ists can provide a "criterion" of invariance that presupposes what it must prove, indeed. So, the accommodation of higher-order entities—that comes with primitive equivalence relations—in a single level ontology results, again, formally impossible from the outset.

In conclusion, my suspicion is that TO-ists do not have available formal resources and further appealing strategies to break these uncertain foundations. A further issue might be whether one-category ontologies can be still coherently regimented once freed from the assumption of a basic and fundamental domain of particulars. In this way, perhaps, it could be possible to restore the nominalist attitude in ontology even if, this time, in a substantively new and revised fashion. This, however, shall be the topic of further studies.

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