3142

ZERO-POINT ANOMALY

José Alexandre Nogueira

and

Adolfo Maia Jr.

Agosto RP 40/93

Relatório de Pesquisa

Instituto de Matemática
Estatística e Ciência da Computação

UNIVERSIDADE ESTADUAL DE CAMPINAS
Campinas - São Paulo - Brasil

ABSTRACT – We study in this work the ambiguity between two definition of vacuum energy, namely, the energy of the Zero-Point fields and the minimum of the Effective Potential, first devised by E. Myers[1]. We name their difference Zero-Point Anomaly (ZPA) and show that for a real scalar field in the geometry of Casimir plates the ZPA there exists but it is undetectable via Casimir forces. We point out possible generalizations of ZPA and physical implications.

IMECC – UNICAMP
Universidade Estadual de Campinas
CP 6065
13081-970 Campinas SP
Brasil

Agosto – 1993

ZERO-POINT ANOMALY

José Alexandre Nogueira

Instituto de Física 'Gleb Wataghin', Universidade Estadual de Campinas, Unicamp, 13083-970 - Campinas - SP - Brasil

Adolfo Maia Jr.

Instituto de Matemática, Universidade Estadual de Campinas, 13.081-970 - Campinas - SP - Brasil.

We study in this work the ambiguity between two definition of vacuum energy, namely, the energy of the Zero-Point fields and the minimum of the Effective Potential, first devised by E. Myers[1]. We name their difference Zero-Point Anomaly (ZPA) and show that for a real scalar field in the geometry of Casimir plates the ZPA there exists but it is undetectable via Casimir forces. We point out possible generalizations of ZPA and physical implications.

Some years ago Eric Myers[1] published a very interesting work on a possible ambiguity of the definition of vacuum energy. He pointed out that we can define vacuum energy as due to Zero-Point fields fluctuations and on the other hand as the minimum of the Effective Potential. Although both definitions coincide for Casimir Effect in electrodynamics, they will give different answers for interacting and/or massive fields. The difference between these two vacuum energies we name Zero-Point Anomaly (ZPA).

In this work we took seriously this ZPA and we calculate it for a general real scalar field (including mass and self-interaction). Surprisingly we found that ZPA there exists (as antecipated by Myers), but it is undetectable via Casimir forces. In other words, the ZPA is a constant, independent on the distance between the Casimir plates, or in more general terms, it is independent on the parameters of the geometry.

We follow, for a easier reading, the notation and definitions of reference 1.

Let $\phi(x)$ be a single real scalar field in a N-dimensional Minikowsky space-time, where $N = m + 1$ and m is the number of spatial dimensions.

The Effective Potential to the first order in the loop expansion (or equivalently in powers of $\hbar$)[2] is given by

$$V_{eff}(\overline{\phi}) = V_{cl}(\overline{\phi}) + \frac{1}{2}\frac{\hbar}{\Omega_M} ln Det \left[\frac{\delta^2 S[\overline{\phi}]}{\delta\phi(x)\delta\phi(y)} \right] \tag{1}$$

where $\overline{\phi} = \langle\phi\rangle$ is the classical field, $S[\phi]$ is the classical Action, Ω_M is the volume of the background space-time manifold and in the Classical Potential $V_{cl}(\phi)$ is included mass and interactions terms. Making the usual analytic continuation to the Euclidian space-time, the classical Action can be written as

$$S[\phi] = \int d^N x [\frac{1}{2}\partial_\mu \phi \partial_\mu \phi + V''_{cl}(\phi)]. \tag{2}$$

From (2) we get the matrix $M(x,y)$ of the quadratic variation of the action $S[\phi]$

$$\mathcal{M}(x,y) \equiv \frac{\delta^2 S[\overline{\phi}]}{\delta\phi(x)\delta\phi(y)} = \delta^N(x-y)[-\delta^{\mu\nu}\partial_\mu\partial_\nu + V''_{cl}(\overline{\phi})]. \tag{3}$$

Now, $\mathcal{M}$ is a elliptical operator (because of the Euclidean analytic continuation) and for these kind of operators we can define the so-called Generalized Zeta Function $\zeta_{\mathcal{M}}(s)$.

Let $\{\lambda_i\}$ the eigenvalües of the operator $\mathcal{M}(x,y)$. The Generalized Zeta Function associated to $\mathcal{M}$ is defined by

$$\zeta_{\mathcal{M}}(s) = \sum_i (\lambda_i)^{-s}. \tag{4}$$

It is well known the relation[3]

$$\ln Det\, \mathcal{M} = -\frac{d\zeta_{\mathcal{M}}(s)}{ds}\bigg|_{s=0} - \ln(2\pi\mu^2)\zeta_{\mathcal{M}}(0), \tag{5}$$

where μ is a unknown parameter (with units of a mass) which should be determined by suplementary renormalizations conditions.

Now, let us consider the Casimir's device in which the field ϕ satisfies Dirichlet boundary conditions on two paralel plates placed at a distance a from each other. In this case, the ζ Function in (4) is given by

$$\zeta_{\mathcal{M}}(s) = \frac{\Omega_M}{a}\sum_{n=0}^{\infty}\int \frac{d^m k}{(2\pi)^m}\left[k^2 + \frac{\pi^2 n^2}{a^2} + V''_{cl}(\overline{\phi})\right]^{-s}. \tag{6}$$

For convenience, we drop the constant Ω_M/a until the formulas (15)-(18) in which we include it again.

Using the integral

$$\int_{-\infty}^{+\infty} (k^2 + A^2)^{-s} d^m k = \frac{\pi^{\frac{m}{2}}\Gamma\left(s - \frac{m}{2}\right)(A^2)^{\frac{m}{2}-s}}{\Gamma(s)}, \tag{7}$$

with $A^2 = \frac{\pi^2 n^2}{a^2} + V''_{cl}(\phi)$, we can write (6) as

$$\zeta_{\mathcal{M}}(s) = \frac{\pi^{m/2}}{(2\pi)^m}\frac{\Gamma\left(s-\frac{m}{2}\right)}{\Gamma(s)}[V''_{cl}(\overline{\phi})]^{\frac{m}{2}-2s}$$
$$+ \frac{\pi^{m/2}}{(2\pi)^m}\left(\frac{\pi}{a}\right)^{m-2s}\frac{\Gamma\left(s-\frac{m}{2}\right)}{\Gamma(s)}\sum_{n=1}^{\infty}\left[n^2+\frac{a^2V''_{cl}(\overline{\phi})}{\pi^2}\right]^{\frac{m}{2}-s} . \quad (8)$$

The sum in the above expression has the form $\sum_{n=1}^{\infty}(n^2+c^2)^{-\mu}$. So, we can use the formula[4]

$$\sum_{n=1}^{\infty}(n^2+c^2)^{-\mu} = -\frac{1}{2}c^{-2\mu} + \frac{\sqrt{\pi}}{2c^{2\mu-1}\Gamma(\mu)}\left[\Gamma\left(\mu-\frac{1}{2}\right)+4\sum_{n=1}^{\infty}(\pi n c)^{\mu-\frac{1}{2}}K_{\mu-\frac{1}{2}}(2\pi n c)\right] . \quad (9)$$

Putting

$$\mu = s-\frac{m}{2} ,$$
$$c^2 = \frac{a^2[V''_{cl}(\overline{\phi})]}{\pi^2}$$

and substituting these values in (9) we get

$$\zeta_{\mathcal{M}}(s) = \frac{1}{2}\frac{\pi^{\frac{m}{2}}}{(2\pi)^m}\frac{\Gamma\left(s-\frac{m}{2}\right)}{\Gamma(s)}[V''_{cl}(\overline{\phi})]^{\frac{m}{2}-s} + a\frac{\pi^{\frac{m-1}{2}}}{2(2\pi)^m}\frac{\Gamma\left(s-\frac{m+1}{2}\right)}{\Gamma(s)}[V''_{cl}(\overline{\phi})]^{\frac{m+1}{2}-s}$$
$$+ a\frac{2\pi^{\frac{m-1}{2}}}{(2\pi)^m}\frac{[V''_{cl}(\overline{\phi})]^{\frac{m+1}{2}-s}}{\Gamma(s)}\sum_{n=1}^{\infty}(na[V''_{cl}(\overline{\phi})]^{1/2})^{s-\frac{m+1}{2}}K_{s-\frac{m+1}{2}}(2na[V''_{cl}(\overline{\phi})]^{\frac{1}{2}}). \quad (10)$$

Now, we proceed to calculate first $V_{eff}(\overline{\phi})$ by (5) and later the Zero-Point Energy E, using the same regularization method. We prefer the ζ Function regularization for its elegance and for later comparison with the Myer's results.

We see that $\zeta_{\mathcal{M}}(s)$ in (10) is a sum of three terms, say

$$\zeta_{\mathcal{M}}(s) = \zeta_1(s)+\zeta_2(s)+\zeta_3(s) \quad (11)$$

Using the analytical properties of modified Bessel Functions $K_\nu(x)$, it is not difficult to show that $\zeta_3(0)=0$. Then

$$\zeta_{\mathcal{M}}(0) = \zeta_1(0)+\zeta_2(0) = \begin{cases} \frac{1}{2}\left(-\frac{V''_{cl}(\overline{\phi})}{4\pi}\right)^{m/2}\frac{1}{\left(\frac{m}{2}\right)!} & \text{if } m \text{ is even} \\ \left(-\frac{V''_{cl}(\overline{\phi})}{4\pi}\right)^{\frac{m+1}{2}}\frac{1}{\left(\frac{m+1}{2}\right)!} & \text{if } m \text{ is odd} \end{cases} \quad (12)$$

From (10) we obtain:
a) for m odd

$$\begin{aligned}
\zeta'_{\mathcal{M}}(0) &= \frac{\pi^{m/2}}{2(2\pi)^m}\Gamma\left(-\frac{m}{2}\right)[V''_{cl}(\bar{\phi})]^{m/2} - \frac{a\pi^{\frac{m-1}{2}}(-1)^{\frac{m+1}{2}}}{2(2\pi)^m\left(\frac{m+1}{2}\right)!}[V''_{cl}(\bar{\phi})]^{\frac{m+1}{2}}\ln(V''_{cl}(\bar{\phi})) \\
&+ a\frac{\pi^{\frac{m-1}{2}}}{2(2\pi)^m}\frac{(-1)^{\frac{m-1}{2}}}{\left(\frac{m+1}{2}\right)!}\left[1+\frac{1}{2}+\cdots+\frac{1}{\frac{m+1}{2}}\right][V''_{cl}(\bar{\phi})]^{\frac{m+1}{2}} \\
&+ a\frac{2\pi^{\frac{m-1}{2}}}{(2\pi)^m}[V''_{cl}(\bar{\phi})]^{\frac{m+1}{2}}\sum_{n=1}^{\infty}\frac{K_{-\frac{m+1}{2}}(2na[V''_{cl}(\bar{\phi})]^{1/2})}{(na[V''_{cl}(\bar{\phi})]^{1/2})^{\frac{m+1}{2}}} \qquad (13)
\end{aligned}$$

b) for m even

$$\begin{aligned}
\zeta'_{\mathcal{M}}(0) &= \frac{a\pi^{\frac{m-1}{2}}}{2(2\pi)^m}\Gamma\left(-\frac{m+1}{2}\right)[V''_{cl}(\bar{\phi})]^{\frac{m+1}{2}} \\
&+ a\frac{2\pi^{\frac{m-1}{2}}}{(2\pi)^m}[V''_{cl}(\bar{\phi})]^{\frac{m+1}{2}}\sum_{n=1}^{\infty}\frac{K_{-\frac{m+1}{2}}(2na[V''_{cl}(\bar{\phi})]^{1/2})}{(na[V''_{cl}(\bar{\phi})]^{1/2})^{\frac{m+1}{2}}} \\
&- \frac{\pi^{m/2}}{2(2\pi)^m}\frac{(-1)^{m/2}}{\left(\frac{m}{2}\right)!}[V''_{cl}(\bar{\phi})]^{m/2}\left\{\ln(V''_{cl}(\bar{\phi})) - \left(1+\frac{1}{2}\cdots+\frac{1}{\frac{m}{2}}\right)\right\}. \qquad (14)
\end{aligned}$$

Substituting (12), (13) and (14) in (5) and after substituting this expression in (1) we obtain the Effective Potential. For Casimir plates at a distance a from each other, it is written as:

a) for m odd

$$\begin{aligned}
V_{eff}(\bar{\phi}) &= V_{cl}(\bar{\phi}) - \frac{\hbar}{2a}\frac{\pi^{m/2}}{2(2\pi)^m}\Gamma\left(-\frac{m}{2}\right)[V''_{cl}(\bar{\phi})]^{m/2} \\
&- \frac{\hbar\pi^{\frac{m-1}{2}}}{(2\pi)^m}[V''_{cl}(\bar{\phi})]^{\frac{m+1}{2}}\sum_{n=1}^{\infty}\frac{K_{-\frac{m+1}{2}}(2na[V''_{cl}(\bar{\phi})]^{\frac{1}{2}})}{(na[V''_{cl}(\bar{\phi})]^{\frac{1}{2}})^{\frac{m+1}{2}}} \\
&+ \frac{\hbar}{2}\frac{\pi^{\frac{m-1}{2}}}{2(2\pi)^m}\frac{(-1)^{\frac{m+1}{2}}}{\left(\frac{m+1}{2}\right)!}\left\{\ln\left(\frac{V''_{cl}(\bar{\phi})}{2\pi\mu^2}\right) - \left(1+\frac{1}{2}+\cdots+\frac{1}{\frac{m+1}{2}}\right)\right\}[V''_{cl}(\bar{\phi})]^{\frac{m+1}{2}} \qquad (15)
\end{aligned}$$

b) for m even

$$
\begin{aligned}
V_{eff}(\bar\phi) &= V_{cl}(\bar\phi) - \frac{\hbar}{2}\frac{\pi^{\frac{m-1}{2}}}{2(2\pi)^m}\Gamma\left(-\frac{m+1}{2}\right)[V''_{cl}(\bar\phi)]^{\frac{m+1}{2}} \\
&- \frac{\hbar\pi^{\frac{m-1}{2}}}{(2\pi)^m}[V''_{cl}(\bar\phi)]^{\frac{m+1}{2}}\sum_{n=1}^{\infty}\frac{K_{-\frac{m+1}{2}}(2na[V''_{cl}(\bar\phi)]^{\frac{1}{2}})}{(na[V''_{cl}(\bar\phi)]^{\frac{1}{2}})^{\frac{m+1}{2}}} \\
&+ \frac{\hbar}{2a}\frac{\pi^{m/2}}{2(2\pi)^m}\frac{(-1)^{m/2}}{\left(\frac{m}{2}\right)!}\left\{\ln\left(\frac{V''_{cl}(\bar\phi)}{2\pi\mu^2}\right) - \left(1+\frac{1}{2}+\cdots+\frac{1}{\frac{m}{2}}\right)\right\}[V''_{cl}(\bar\phi)] \quad (16)
\end{aligned}
$$

The term independent on a in Eq(s) (15) and (16) corresponds to the Effective Potential for the unconstrained field, namely:

a) for m odd

$$
V^0_{eff}(\bar\phi) = V_{cl}(\bar\phi) + \frac{\hbar}{2}\frac{\pi^{\frac{m-1}{2}}}{2(2\pi)^m}\frac{(-1)^{\frac{m+1}{2}}}{\left(\frac{m+1}{2}\right)!}[V''_{cl}(\bar\phi)]^{\frac{m+1}{2}}\left\{\ln\left(\frac{V''_{cl}(\bar\phi)}{2\pi\mu^2}\right) - \left(1+\frac{1}{2}+\cdots+\frac{1}{\frac{m+1}{2}}\right)\right\} \quad (17)
$$

b) for m even

$$
V^0_{eff}(\bar\phi) = V_{cl}(\bar\phi) - \frac{\hbar}{2}\frac{\pi^{\frac{m-1}{2}}}{2(2\pi)^m}\Gamma\left(-\frac{m+1}{2}\right)[V''_{cl}(\bar\phi)]^{\frac{m+1}{2}} \quad (18)
$$

Now, as is well known, the physical (renormalized) vacuum energy is given by the usual subtracting procedure. We have

a) for m odd

$$
\begin{aligned}
V^R_{eff} \equiv V_{eff} - V^0_{eff} &= -\frac{\hbar}{2a}\frac{\pi^{m/2}}{2(2\pi)^m}\Gamma\left(-\frac{m}{2}\right)[V''_{cl}(\bar\phi)]^{m/2} \\
&- \frac{\hbar\pi^{\frac{m-1}{2}}}{(2\pi)^m}[V''_{cl}(\bar\phi)]^{\frac{m+1}{2}}\sum_{n=1}^{\infty}\frac{K_{-\frac{m+1}{2}}(2na[V''_{cl}(\bar\phi)]^{1/2})}{(na[V''_{cl}(\bar\phi)]^{1/2})^{\frac{m+1}{2}}} \quad (19)
\end{aligned}
$$

b) for m even

$$
\begin{aligned}
V^R_{eff} &= \frac{\hbar}{2a}\frac{\pi^{m/2}}{2(2\pi)^m}\frac{(-1)^{m/2}}{\left(\frac{m}{2}\right)!}\left\{\ln\left(\frac{V''_{cl}(\bar\phi)}{2\pi\mu^2}\right) - \left(1+\frac{1}{2}+\cdots+\frac{1}{\frac{m}{2}}\right)\right\}[V''_{cl}(\bar\phi)]^{m/2} \\
&- \frac{\hbar\pi^{\frac{m-1}{2}}}{(2\pi)^m}[V''_{cl}(\bar\phi)]^{\frac{m+1}{2}}\sum_{n=1}^{\infty}\frac{K_{-\frac{m+1}{2}}(2na[V''_{cl}(\bar\phi)]^{1/2})}{(na[V''_{cl}(\bar\phi)]^{1/2})^{\frac{m+1}{2}}}. \quad (20)
\end{aligned}
$$

On the other hand, we have the familiar Zero-Point Energy

$$E = \sum_i \frac{\hbar}{2} w_i. \tag{21}$$

For the Casimir plates the w_i are given by

$$w_i = \left[p^2 + \frac{\pi^2 n^2}{a^2} + V''_{cl}(\phi_0)\right]^{1/2} \tag{22}$$

where ϕ_0 is the point of minimum for the Classical Potential $V(\phi)$.

If the plates have a surface with area L, then (21) and (22) give for the Zero-Point Energy

$$E = \frac{\hbar}{2} \sum_{n=0}^{\infty} \int_{-\infty}^{+\infty} L^{m-1} \left(p^2 + \frac{\pi^2 n^2}{a^2} + V''_{cl}(\phi_0)\right)^{1/2} \frac{d^{m-1}p}{(2\pi)^{m-1}}. \tag{23}$$

Again, we define the ζ Function associated to (23) as

$$\zeta_H(\mu) = \sum_{n=0}^{\infty} \int_{-\infty}^{+\infty} [p^2 + \frac{n^2\pi^2}{a^2} + V''_{cl}(\phi_0)]^{-\mu} \frac{d^{m-1}p}{(2\pi)^{m-1}} \tag{24}$$

where the subscript H at the ζ Function means that w_i are eingenvalues of the Hamiltonian operator.

The Zero-Point Energy is, then, given by

$$E = \frac{\hbar}{2} \lim_{\mu \to -\frac{1}{2}} [\zeta_H(\mu) L^{m-1}]. \tag{25}$$

Substituting (24) in (25) we find the ZPE for plates at a distance a,

$$\begin{aligned} E = & - \frac{\hbar}{2} \frac{L^{m-1} \pi^{m/2}}{2(2\pi)^m} \Gamma\left(-\frac{m}{2}\right) [V''_{cl}(\phi_0)]^{m/2} \\ & - \frac{\hbar}{2} L^{m-1} \frac{\pi^{\frac{m-1}{2}}}{2(2\pi)^m} \Gamma\left(-\frac{m+1}{2}\right) a [V''_{cl}(\phi_0)]^{\frac{m+1}{2}} \\ & - \hbar L^{m-1} \frac{\pi^{\frac{m-1}{2}}}{(2\pi)^m} a [V''_{cl}(\phi_0)]^{\frac{m+1}{2}} \sum_{n=1}^{\infty} \frac{K_{-\frac{m+1}{2}}(2na[V''_{cl}(\phi_0)]^{1/2})}{(na[V''_{cl}(\phi_0)]^{1/2})^{\frac{m+1}{2}}}. \end{aligned} \tag{26}$$

Again for the free field at the same volume $\Omega = L^{m-1}\, a$, the ZPE is given by

$$E_0 = -\frac{\hbar}{2} L^{m-1} a \frac{\pi^{\frac{m-1}{2}}}{2(2\pi)^m} \Gamma\left(-\frac{m+1}{2}\right) [V''_{cl}(\phi_0)]^{\frac{m+1}{2}}. \tag{27}$$

The physical (renormalized) Zero-Point Energy density is defined by the usual subtraction

$$\varepsilon^R = \frac{E - E_0}{L^{m-1}a}. \tag{28}$$

Substituting (26) and (27) in (28) we get

$$\begin{aligned}\varepsilon^R = & -\frac{\hbar}{2a}\frac{\pi^{m/2}}{2(2\pi)^m}\Gamma\left(\frac{-m}{2}\right)[V''_{cl}(\phi_0)]^{m/2} \\ & -\frac{\hbar\pi^{\frac{m-1}{2}}}{(2\pi)^m}[V''_{cl}(\phi_0)]^{\frac{m+1}{2}}\sum_{n=1}^{\infty}\frac{K_{-\frac{m+1}{2}}(2na[V''_{cl}(\phi_0)]^{1/2})}{(na[V''_{cl}(\phi_0)^{1/2}])^{\frac{m+1}{2}}}.\end{aligned} \tag{29}$$

Now, we define the renormalized Zero-Point Anomaly $\mathcal{A}^R$ as the difference between the Effective Potential ((19) and (20)) and the Zero-Point Energy density (29)

$$\mathcal{A}^R = V^R_{eff} - \varepsilon^R \tag{30}$$

and we obtain

$$\text{a) } \mathcal{A}^R = 0 \quad \text{for } m \text{ odd}, \tag{31}$$

$$\begin{aligned}\text{b) } \mathcal{A}^R = & \frac{\hbar}{2a}\frac{\pi^{m/2}}{2(2\pi)^m}\frac{(-1)^{m/2}}{\left(\frac{m}{2}\right)!}\left\{\ln\left(\frac{V''_{cl}(\overline{\phi})}{2\pi\mu^2}\right) - \left(1 + \frac{1}{2} + \cdots + \frac{1}{\frac{m}{2}}\right)\right\}[V''_{cl}(\overline{\phi})]^{m/2} \\ & + \frac{\hbar}{2a}\frac{\pi^{m/2}}{2(2\pi)^m}\Gamma\left(-\frac{m}{2}\right)[V''_{cl}(\overline{\phi})]^{m/2} \quad \text{for } m \text{ even}.\end{aligned} \tag{32}$$

In (31) and (32) we have used the aproximation $\overline{\phi} \cong \phi_0$. This is correct in first aproximation in $\hbar$ and therefore (31) and (32) are the ZPA with quantum corrections in first order in $\hbar$, as we can see from above analysis.

Also observe that the ZPA there exists only in even dimensions but, even in this case, Casimir forces (atraction between two plates) will not able to distinguishe between the two definitions above, that is, there will not be any detectable shift in the Casimir Energy. This is because $\mathcal{A}^R$ in (30) is proportional to a^{-1} and then the shift in the Casimir pressure is given by

$$\Delta F = -\frac{\partial}{\partial a}\left(a\mathcal{A}^R\right) = 0. \tag{33}$$

The important point in this work is that ZPA is a new concept about vacuum energy that depends on the experimental apparatus, or in other words, it depends crucially (as the sign of Casimir Energy) on the geometry. In this sense, it would be interesting to search for a geometry in which the ZPA is nonvanishing.

RELATÓRIOS DE PESQUISA — 1993

01/93 **On the Convergence Rate of Spectral Approximation for the Equations for Nonhomogeneous Asymmetric Fluids** — *José Luiz Boldrini* and *Marko Rojas-Medar.*

02/93 **On Fraisse's Proof of Compactness** — *Xavier Caicedo* and *A. M. Sette.*

03/93 **Non Finite Axiomatizability of Finitely Generated Quasivarieties of Graphs** — *Xavier Caicedo.*

04/93 **Holomorphic Germs on Tsirelson's Space** — *Jorge Mujica* and *Manuel Valdivia.*

05/93 **Zitterbewegung and the Electromagnetic Field of the Electron** — *Jayme Vaz Jr.* and *Waldyr A. Rodrigues Jr.*

06/93 **A Geometrical Interpretation of the Equivalence of Dirac and Maxwell Equations** — *Jayme Vaz Jr.* and *Waldyr A. Rodrigues Jr.*

07/93 **The Uniform Closure of Convex Semi-Lattices** — *João B. Prolla.*

08/93 **Embedding of Level Continuous Fuzzy Sets and Applications** — *Marko Rojas-Medar, Rodney C. Bassanezi* and *Heriberto Román-Flores.*

09/93 **Spectral Galerkin Approximations for the Navier-Stokes Equations: Uniform in Time Error Estimates** — *Marko A. Rojas-Medar* and *José Luiz Boldrini.*

10/93 **Semigroup Actions on Homogeneous Spaces** — *Luiz A. B. San Martin* and *Pedro A. Tonelli.*

11/93 **Clifford Algebra Approach to the Barut-Zanghi Model as a Hamiltonian System** — *Jayme Vaz Jr.* and *Waldyr A. Rodrigues Jr.*

12/93 **Propagation of Scalar Waves in Layered Media** — *Lúcio Tunes dos Santos* and *Martin Tygel.*

13/93 **On the Convergence of the NMO-Power Series for a Horizontally Stratified Medium** — *Martin Tygel.*

14/93 **Convergence Rates in the Sobolev H^s-Norm of Approximations by Discrete Convolutions** — *Sônia M. Gomes.*

15/93 **On the Choice of the Space Dimension in Ill-Posed Problems** — *Cristina Cunha.*

16/93 **Elliptic Equations in R^2 with Non-linearities in the Critical Range** — *D. G. de Figueiredo, O. H. Miyagaki* and *B. Ruf.*

17/93 **Drug Kinetics and Drug Resistance in Optimal Chemotherapy** — *M. I. S. Costa, J. L. Boldrini* and *R. C. Bassanezi.*

18/93 **Chemotherapeutic Treatments Involving Drug Resistance and Level of Normal Cells as a Criterion of Toxicity** — *M. I. S. Costa, J. L. Boldrini* and *R. C. Bassanezi.*

19/93 **Bifurcation of Singularities of Reversible Systems** — *Marco Antonio Teixeira.*

20/93 Sistemas Não Lineares e Fractais — *Lúcio Tunes dos Santos.*

21/93 New Integral Representation of the Solution of Schrödinger Equation with Arbitrary Potential — *Rodolfo L. Monaco* and *Waldyr A. Rodrigues Jr.*

22/93 The Module of Derivations of a Stanley–Reisner Ring — *Paulo Brumatti* and *Aron Simis.*

23/93 On the Convergence Rate of Spectral Approximation for the Equations for Chemical Active Fluid — *Marko Rojas-Medar* and *Sebastián A. Lorca.*

24/93 Sufficient Conditions for Minima of some Translation Invariant Functionals — *Orlando Lopes.*

25/93 A Constrained Minimization Problem with Integrals on the Entire Space — *Orlando Lopes.*

26/93 O Pensamento Reducionista na Filosofia de Niels Bohr — *José Emílio Maiorino.*

27/93 On the first curve of the Fučik spectrum of an elliptic operator — *D.G. de Figueiredo* and *J.-P. Gossez.*

28/93 Generalização dos Testes de Shirley e de House — *Belmer Garcia Negrillo.*

29/93 Compacidad y Compactificación en Teoría de Modelos — *J. C. Cifuentes.*

30/93 Global Strong Solutions of the Equations for the Motion of Nonhogeneous Incompressible Fluids — *José Luiz Boldrini* and *Marko Rojas-Medar.*

31/93 A Equação de Laplace no Universo de de-Sitter-Castelnuovo — *D. Gomes* e *E. Capelas de Oliveira.*

32/93 Klein-Gordon Wave Equation in the de Sitter Universe — *E. Capelas de Oliveira* and *E.A. Notte Cuello.*

33/93 Mittag-Leffler Methods in Analysis — *Jorge Mujica.*

34/93 The Initial Value Problem for a Generalized Boussinesq Model — *Sebastián A. Lorca* and *José Luiz Boldrini.*

35/93 Problemas Minimax e Aplicações — *José Mario Martínez, Lúcio Tunes dos Santos* e *Sandra Augusta Santos.*

36/93 An Extension of the Theory of Secant Preconditioners — *José Mario Martínez.*

37/93 Convergence Estimates for the Wavelet-Galerkin Method: Superconvergence at the Node Points — *Sônia M. Gomes.*

38/93 An Error Estimate Uniform in Time for Spectral Semi-Galerkin Approximations of the Nonhomogeneous Navier-Stokes Equations — *J. L. Boldrini* and *M. Rojas-Medar.*

39/93 More About the Time Analysis of Tunnelling Processes — *Vladislav S. Olkhovsky* and *Erasmo Recami.*