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Impact of a magnetic field on the thermodynamics of magnetized quark matter

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Abstract. We investigate the effect of a magnetic field on the thermodynamics of magnetized quark matter at finite temperature. By using the Nambu Jona–Lasino (NJL) model, we show that the lattice results for the quark consensate can be reproduced when the coupling constant G of the model decreases with the magnetic field and the temperature. Our results show that thermodynamic quantities and quark condensates are very sensitive to the dependence of G with the temperature, even in the absence of a magnetic field.

1. Introduction

Motivated by the fact that strong magnetic fields may be produced in noncentral heavy-ion collisions [1, 2], investigations of the effects of a magnetic field on the phase diagram of strongly interacting matter became a subject of great interest in recent years.

At zero temperature, the great majority of effective models for Quantum Chromodynamics (QCD) are in agreement with respect to the occurrence of Magnetic Catalysis (MC) effect, in that the quark condensate increases with the magnetic field B. On the other hand, finite temperature lattice QCD simulations show that the pseudo-critical temperature T_{pc} for chiral symmetry restoration decreases as the strength of the magnetic field increases, a phenomenon known as Inverse Magnetic Catalysis (IMC) [3]. The great majority of effective models fail to explain this phenomenon — see Ref. [4] for a recent review.

A large body of work has been devoted to the search for possible mechanisms responsible for the IMC. In a recent work by some of us [5], we proposed that the IMC found by lattice simulations can be explained within the NJL model if the coupling constant G decreases with both the magnetic field and the temperature. A similar mechanism was proposed within $SU(3)$ PNJL models, but with G depending only on the magnetic field [6]. Another possible explanation was provided within the framework of holography [7]. In the present work we refine the running of the coupling $G(eB, T)$ quantitatively and reinvestigate the effects of the magnetic field on thermodynamic quantities of the magnetized quark matter within the $SU(2)$ NJL model.

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2. The magnetized NJL model

We use the SU(2) version of the NJL model [8], defined by the Lagrangian density

$$\mathcal{L}_{\rm NJL} = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi + G\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}\psi)^2\right] . \tag{1}$$

The field ψ represents a flavor iso-doublet of u and d quark flavors and N_c-plet of quark fields, $\vec{\tau}$ are isospin Pauli matrices. Since the model is non-renormalizable, we need to specify some regularization scheme; here we use a sharp momentum cutoff Λ as an ultraviolet regulator. Therefore, Λ, G and the quark masses $m_f, f = u, d$ are free parameters which are fixed [9, 10] by fitting the values of the pion mass m_π, pion decay constant f_π and quark condensate $\langle\bar{\psi}_f\psi_f\rangle$.

Thermodynamic quantities are computed from the grand thermodynamical potential in the mean field approximation — see e.g. Ref. [10] for details:

$$\Omega^{\rm NJL} = \frac{(M-m)^2}{4G} + \frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\ln[-p^2+M^2] . \tag{2}$$

In order of include the effects of T and B simultaneously in the thermodynamic potential, it is usual to make use of the following transformations [11]:

$$p_0 \rightarrow i(\omega_n - i\mu) , \tag{3}$$

$$p^2 \rightarrow p_z^2 + (2k+1-s)|q_f|B , \tag{4}$$

$$\int_{-\infty}^{+\infty}\frac{d^4p}{(2\pi)^4} \rightarrow i\frac{T|q_f|B}{2\pi}\sum_{n=-\infty}^{\infty}\sum_{k=0}^{\infty}\int_{-\infty}^{+\infty}\frac{dp_z}{2\pi} , \tag{5}$$

with $s = \pm 1$, $k = 0, 1, 2, \cdots$. The Matsubara frequencies for fermions are given by $\omega_n = (2n+1)\pi T$, with $n = 0, \pm 1, \pm 2, \cdots$ and k represents the Landau levels. After these replacements, the grand potential including the effects of the magnetic field can be written as

$$\Omega^{\rm NJL} = \frac{(M-m)^2}{4G} + \Omega^{\rm NJL}_{\rm vac} + \Omega^{\rm NJL}_{\rm mag} + \Omega^{\rm NJL}_{\rm med} , \tag{6}$$

where

$$\Omega^{\rm NJL}_{\rm vac} = -2N_cN_f\int\frac{d^3p}{(2\pi)^3}\sqrt{p^2+M^2} , \tag{7}$$

and

$$\Omega^{\rm NJL}_{\rm mag} = -\sum_{f=u}^{d}\frac{N_c(|q_f|B)^2}{2\pi^2}\left\{\zeta'[-1,x_f] - \frac{1}{2}[x_f^2 - x_f]\ln(x_f) + \frac{x_f^2}{4}\right\} , \tag{8}$$

with $x_f = M^2/(2|q_f|B)$, $\zeta'[-1,x_f] = d\zeta(z,x_f)/dz|_{Z=-1}$ and $\zeta(z,x_f)$ representing the Hurwitz zeta function. The last term in Eq. (6), which contains the effects of temperature and magnetic field, is given by

$$\Omega^{\rm NJL}_{\rm med} = -\frac{N_c}{2\pi}\sum_{f=u}^{d}\sum_{k=0}^{\infty}\alpha_k(|q_f|B)\int_{-\infty}^{+\infty}\frac{dp_z}{2\pi} \times \left\{T\ln[1+e^{-[E_{p,k}(B)+\mu]/T}] + T\ln[1+e^{-[E_{p,k}(B)-\mu]/T}]\right\} , \tag{9}$$

where $E_{p,k}(B) = \sqrt{p_z^2 + 2k|q_f|B + M^2}$ and $\alpha_k = 2 - \delta_{0k}$. More details in the derivation of these expressions can be found in Refs. [11, 12]. The effective quark mass M_f is obtained in a self-consistent way by solving the gap equation

$$M_f = m_f - 2G\sum_f\langle\bar{\psi}_f\psi_f\rangle , \tag{10}$$

where $\langle\bar{\psi}_f\psi_f\rangle$ represents the quark condensate of flavour f. The explicit expression for the quark condensate at finite T and B is given by

$$\begin{aligned}\langle\bar{\psi}_f\psi_f\rangle &= -\frac{N_c M}{2\pi^2}\left\{\Lambda\sqrt{\Lambda^2+M^2}-\frac{M^2}{2}\ln\left[\frac{(\Lambda+\sqrt{\Lambda^2+M^2})^2}{M^2}\right]\right\}\\ &-\frac{N_c M}{2\pi^2}|q_f|B\left\{\ln[\Gamma(x_f)]-\frac{1}{2}\ln(2\pi)+x_f-\frac{1}{2}(2x_f-1)\ln(x_f)\right\}\\ &+\frac{N_c M}{2\pi^2}\sum_{k=0}^{\infty}\alpha_k|q_f|B\int_{-\infty}^{\infty}\frac{dp_z}{E_{p,k}(B)}\left\{\frac{1}{e^{[E_{p,k}(B)]/T}+1}\right\}\ . \end{aligned} \tag{11}$$

It is important to note that we are working with Gaussian natural units where $1\,\mathrm{GeV}^2 \simeq 1.44\times10^{19}$ G and $|q_f|$ is the absolute value of the quark electric charge. At finite magnetic fields, the quark condensate for the flavors u and d are different due to their different electric charges ($|q_u| = 2e/3$, $|q_d| = e/3$, $e = 1/\sqrt{137}$).

An interesting feature that occurs in the two flavour case is that, in principle, one should have two coupled gap equations for the two distinct flavors, but in this case the different condensates contribute to M_u and M_d in a symmetric way and, since we are using $m_u = m_d = m$, one can write $M_u = M_d = M$ (more details see Ref. [11]).

3. Fitting NJL coupling constant with the lattice results

In this section we describe the fitting procedure used to obtain the thermomagnetic dependence of the NJL coupling constant. We fit the NJL coupling $G(eB,T)$ so that the NJL model reproduces the average of the quark condensates, $(\Sigma_u+\Sigma_d)/2$, obtained by the lattice simulation of Ref. [3]. The lattice results of Ref. [3] are for zero temperature and high temperatures, with no data points between $T = 0$ and $T = 113$ MeV. Given the fact that the discrepancies between lattice results and effective models appear in the region where chiral symmetry is partially restored (crossover), we fit the NJL coupling constant with the lattice data in this regime and then extrapolate our results to zero temperature.

We can get a good fit to the lattice data for the average of the quark condensates, $(\Sigma_u+\Sigma_d)/2$, using the following equation for NJL coupling constant:

$$G(eB,T) = c\left[1-\frac{1}{1+\exp\left[d\,(T_a-T)\right]}\right]+s, \tag{12}$$

where the parameters c, s, β and T_a depend on eB. For zero magnetic field, the values of the fitting parameters are: $c = 0.9$ GeV^{-2}, $s = 3.7311$ GeV^{-2}, $d = 0.4$ MeV^{-1}, and $T_a = 168$ MeV. The extrapolation of our fit to $T = 0$ gives $G(0,0) = 4.6311$ GeV^{-2}. Using $\Lambda = 0.650$ GeV and $m_0 = 5.5$ MeV, we obtain $f_\pi = 84.4498\,\mathrm{MeV}$, $m_\pi = 142.177\,\mathrm{MeV}$ and $\langle\bar{\psi}_f\psi_f\rangle^{1/3} = -236.374\,\mathrm{MeV}$ in the vacuum.

In panel (a) of Fig. 1 we compare the temperature dependence of average of the condensates $(\Sigma_u+\Sigma_d)/2$ at zero magnetic field obtained with the NJL model with a coupling G temperature independent (solid black line) and with $G(0,T)$ (red dashed line) given by Eq. (12). In the transition region, around $T \simeq 170$ MeV, one sees that the difference between the curves is quite substantial. In panel (b) of the same figure, we show the running of the G as given by Eq. (12) for zero magnetic field.

In the next section, we investigate the impact on thermodynamic quantities calculated with the NJL model when using a running $G(eB,T)$ as given in Eq. (12).

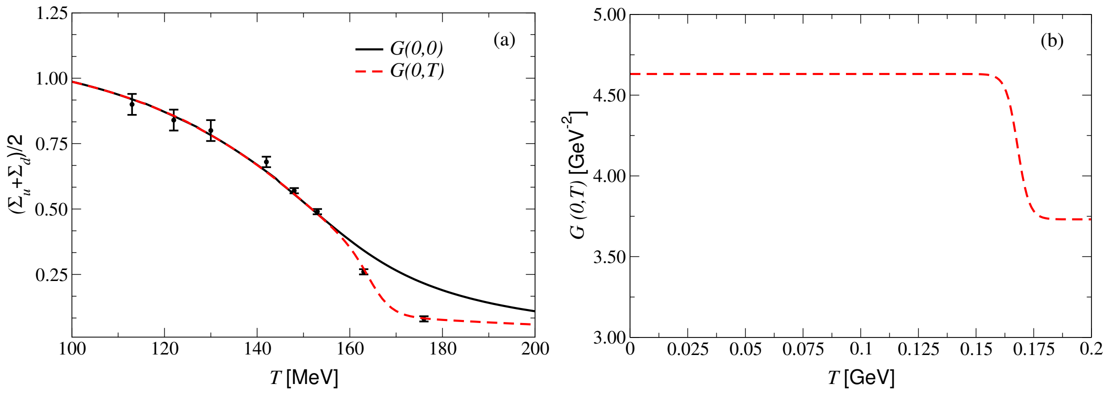

Figure 1. Panel (a): Condensate average as a function of the temperature at zero magnetic field, where data points are from the lattice simulations of Ref. [3]. Panel (b): NJL coupling constant $G(eB, T)$ at zero magnetic field.

4. Numerical Results

We start examining the effects of the running G with the temperature in the quark mass M and in quark condensate $\langle\bar{\psi}_f \psi_f\rangle$ — for zero magnetic field, the u and d condensates are equal (recall we are using $m_u = m_d$). We can observe in Fig. 2 that in the critical region these quantities are very sensitive to the running with T and at high temperatures the model displays a crossover (where chiral symmetry is partially restored) similar to the case with no running G. The difference is that the drop in the mass and in the condensate is much sharper when we let the coupling run with the temperature.

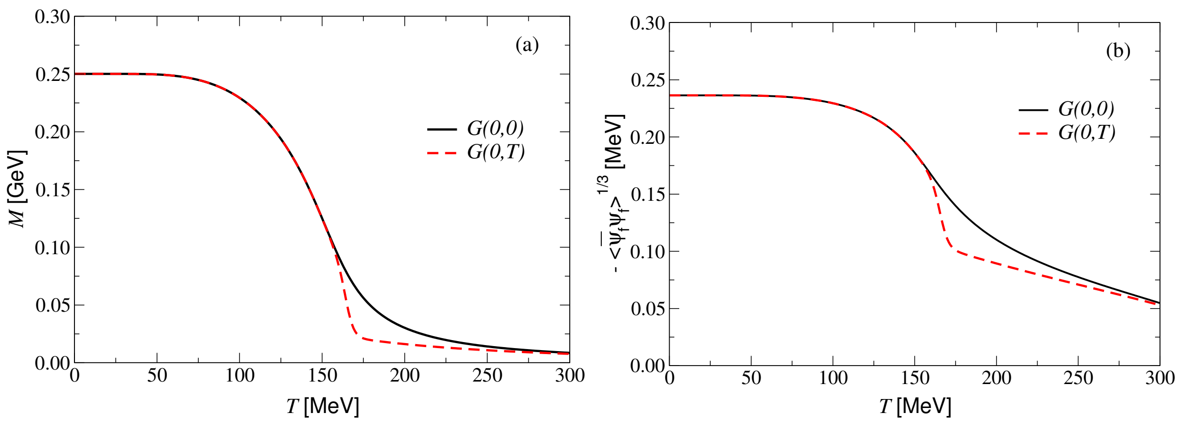

Figure 2. Temperature dependence of the constituent quark mass M (panel (a)) and the quark condensate (panel (b)).

The model displays a crossover and the pseudo-critical temperature depends on the observable used to locate the transition. Here we use the peak of the thermal susceptibility, which is given by

$$\chi_T = -m_\pi \frac{\partial \sigma}{\partial T}, \quad \text{where} \quad \sigma = \frac{\langle\bar{\psi}_u \psi_u\rangle(B,T) + \langle\bar{\psi}_d \psi_d\rangle(B,T)}{\langle\bar{\psi}_u \psi_u\rangle(B,0) + \langle\bar{\psi}_d \psi_d\rangle(B,0)} \,. \tag{13}$$

Other quantities of interest are pressure, the negative of the free energy, $p = -\Omega(M, eB, T)$, the entropy density, obtained from $s = dp/dT$, and the energy density $\varepsilon = -p + T\,s + \mu\,\rho$. We normalize the pressure by subtracting the pressure at zero temperature, $p(eB, 0)$ so that the normalized pressure, p_N, is given by

$$p_N(eB, T) = p(eB, T) - p(eB, 0)\ . \tag{14}$$

In addition, we also consider the interaction measure, speed of sound, and specific heat, given respectively by

$$\Delta = \frac{\varepsilon - 3p_N}{T^4}, \qquad c_s^2 = \left(\frac{\partial p_N}{\partial \varepsilon}\right)_v, \qquad c_v = \left(\frac{\partial \varepsilon}{\partial T}\right)_v . \tag{15}$$

In panel (a) of Fig. 3, we show the thermal susceptibility defined by Eq. (13) as a function of the temperature at zero magnetic field. The result indicates that the running of coupling with the temperature change the position of the χ_T peak and T_{pc} is slightly larger if we compare with the case with no running. The interaction measure Δ is shown in panel (b) of the figure.

We next examine the effect of the running of $G(eB = 0, T)$ with the temperature in all relevant thermodynamic quantities.

In Fig. 4 we show our results for the scaled pressure p_N/T^4, scaled energy density ε/T^4, the equation of state (eos) p_N/ε and scaled entropy density s/T^3. In panel (a) we see that the running $G(0, T)$ does not affect the behaviour of the scaled pressure and in panel (b) we observe the influence of the $G(0, T)$ in the scaled energy density that also will appear in the critical region of the eos shown in panel (c). In panel (d) we show that the scaled entropy is not affected by the running.

Fig. 5 displays the temperature dependence of the speed of sound squared c_s^2, scaled specific heat c_v/T^3. Note that c_s^2 and c_v display different behaviours compared to the case with no running. With the running, c_s^2 shows a downward cusp in T_{pc} while c_v/T^3 shows a upward cusp in T_{pc} as expected.

In figure 6 we present our results for the condensate average as a function of temperature for different values of the magnetic field. More details on the fit of $G(eB, T)$, the parametrization and their effect on the thermodynamic quantities of magnetized quark matter will be discussed in a forthcoming work [13].

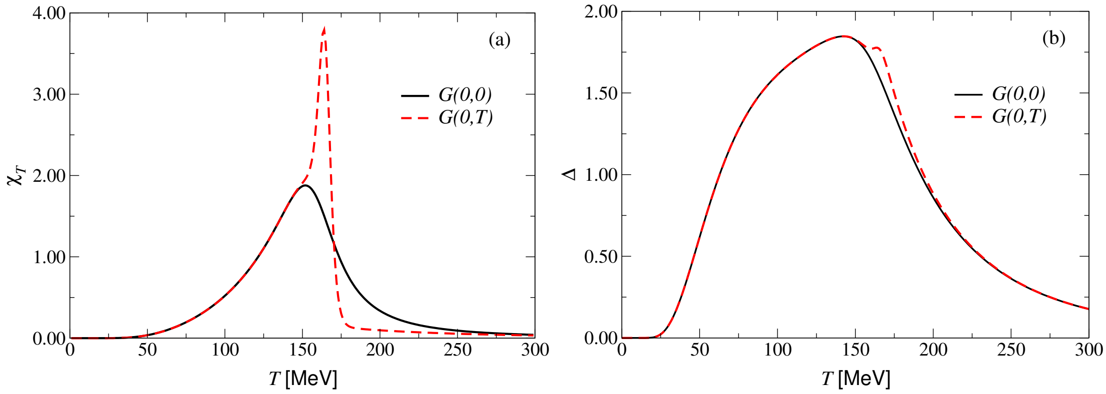

Figure 3. Temperature dependence of the normalized thermal susceptibility χ_T (panel (a)) and interaction measure (panel(b)) for zero magnetic field.

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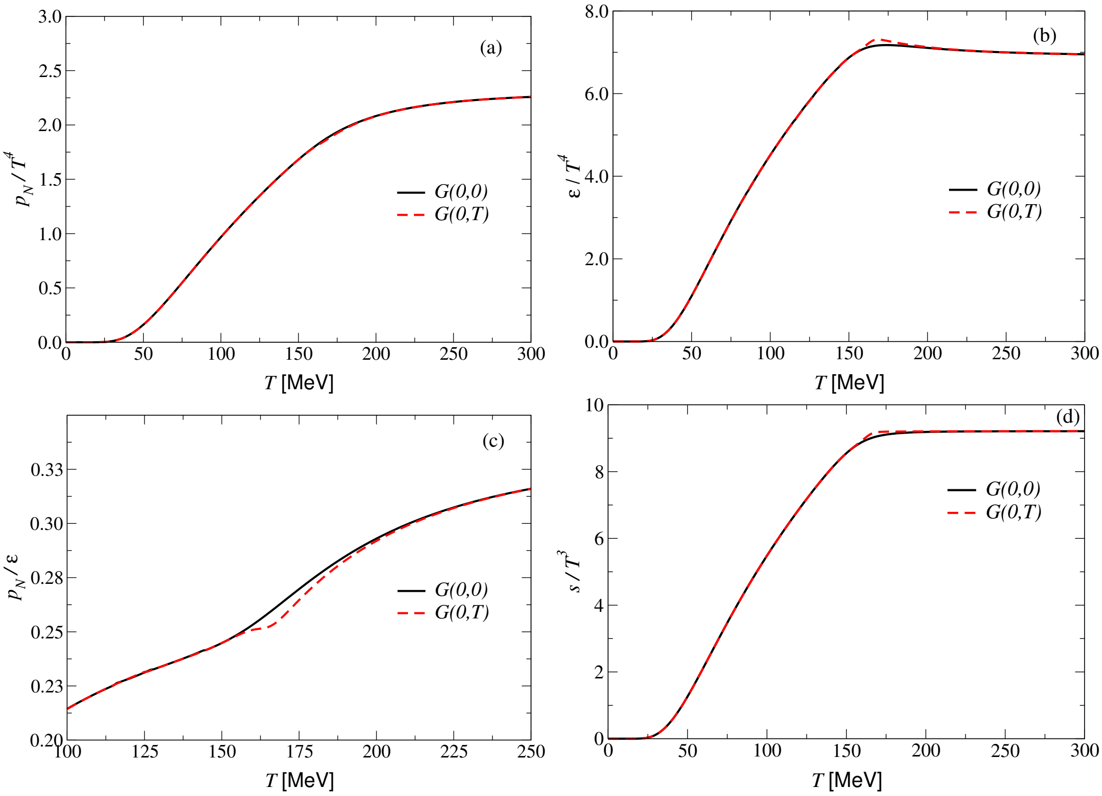

Figure 4. Panel (a): Scaled pressure p_N/T^4. Panel (b): Scaled energy density ε/T^4. Panel (c): Equation of state p_N/ε. Panel (d): Scaled entropy density $s/T^3(s = dP/dT)$.

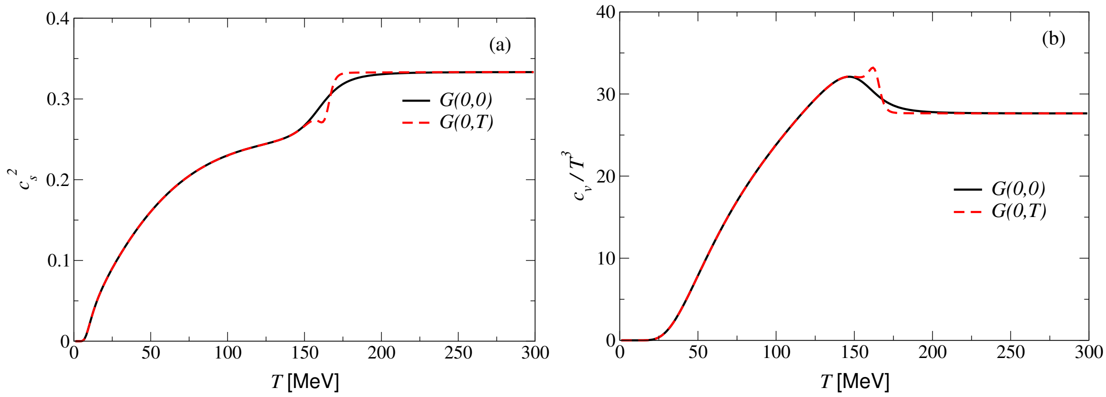

Figure 5. Temperature dependence of speed of sound squared (panel (a)) and scaled specific heat (panel (b)) for zero magnetic field.

5. Final Remarks

In this work we have considered the two flavor NJL model for hot and magnetized quark matter within the mean field approximation. At vanishing magnetic fields, we have shown that lattice QCD results can be reproduced in the NJL model if the coupling constant has a temperature dependence. We have analysed the effects of the running $G = G(0,T)$ on the quark masses, quark condensates as well as on thermodynamic quantities. We have improved a previous attempt in

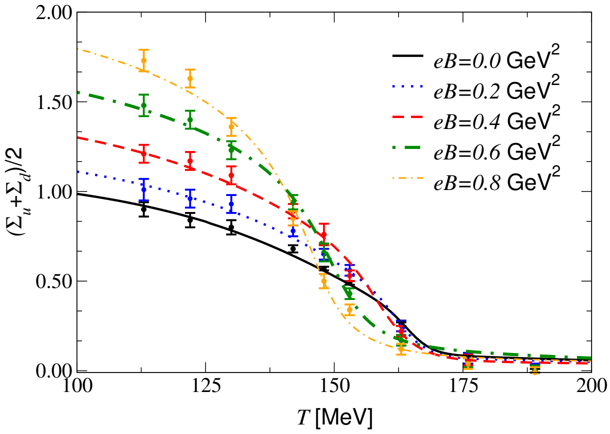

Figure 6. The condensate average as a function of the temperature. Data points are from the lattice simulations of Ref. [3]. The curves were obtained using our fit of $G(eB,T)$[13].

Ref. [5] and fitted the NJL coupling constant so that the model reproduces the average of quark condensates computed on the lattice [3].

Our numerical results show that this running with T introduces modifications in the behaviour of the quark condensate, quark mass and in the thermodynamic properties of the system. The equation of state and thermodynamic quantities like the sound velocity squared and the specific heat are very sensitive to this running in the region around T_{pc}. At finite magnetic fields and temperatures we have presented results for the average of quark condensates that are in good agreement with lattice QCD simulations of Ref. [3]. A detailed investigation of the effect of the running with both T and B on the thermodynamics of magnetized quark matter is in progress [13].

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