

IGOR MACIEL DE OLIVEIRA E SILVA

"METHODOLOGY FOR COOLING WATER SYSTEMS DESIGN"

"METODOLOGIA PARA PROJETO DE SISTEMAS DE ÁGUA DE RESFRIAMENTO"

CAMPINAS 2014



UNIVERSIDADE ESTADUAL DE CAMPINAS Faculdade de Engenharia Química

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"METHODOLOGY FOR COOLING WATER SYSTEMS DESIGN"

"METODOLOGIA PARA PROJETO DE SISTEMAS DE ÁGUA DE RESFRIAMENTO"

Thesis presented to the School of Chemical Engineering of the University of Campinas in partial fulfilment of the requirements for the Master's degree in Chemical Engineering.

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Supervisor/Orientador: Dr. ROGER JOSEF ZEMP

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"Live as if you were to die tomorrow. Learn as if you were to live forever."

Mahatma Gandhi

ABSTRACT

Cooling water systems are the most common method of waste heat disposal in industry. Conventional recirculating cooling water systems have a heat exchanger network in a parallel arrangement, demanding not only substantial cooling water recirculation, but also large cooling towers. Although cooling water reuse reduces the amount of water that is recirculated in the system, thereby increasing the cooling tower capacity and performance, the pressure drop in the heat exchanger network may significantly increase due to series-parallel arrangements. This study introduces a methodology to design different cooling water systems and to analyse the cooling water reuse impacts on the heat exchanger network pressure drop and on the cooling tower size. From a superstructure model, a combinatorial algorithm in conjunction with the optimisation tool Solver in Microsoft Excel is used to solve a nonlinear problem for each heat exchanger network structure. Pressure drop in heat exchanger networks is evaluated by a methodology that is based on Graph Theory and that uses topological sorting and critical path algorithms. Merkel's method is used to model the cooling tower height and to assess the required cooling tower volume for each heat exchanger network. A case study is used to illustrate each step as the methodology is developed, aiming to provide a basis for a conceptual stage during the cooling water system design.

Key Words: Process Integration, Cooling Water System, Heat exchanger network, Pressure drop

RESUMO

Sistemas de água de resfriamento são o método mais comum de rejeição de calor na indústria. Sistemas convencionais de água de resfriamento recirculante possuem uma rede de trocadores de calor em uma configuração paralela, demandando grande quantidade de recirculação de água e torres de resfriamento. Embora a reutilização de água de resfriamento reduza a quantidade de água que é necessária no sistema e aumente o desempenho e capacidade da torre de resfriamento, a queda de pressão na rede de trocadores de calor pode aumentar devido ao seu arranjo em série-paralelo. Este estudo introduz uma metodologia para projetar diferentes sistemas de água de resfriamento e para analisar os impactos da reutilização de água sobre a queda de pressão na rede de trocadores de calor e sobre a torre de resfriamento. A partir de um modelo de super-estrutura, utiliza-se um algoritmo combinatorial com o auxílio da ferramenta de otimização Solver do Microsoft Excel para resolver um problema não-linear (NLP) de cada estrutura de rede de trocadores de calor. A queda de pressão em redes de trocadores de calor é avaliada por uma metodologia baseada na Teoria dos Grafos e utiliza os algoritmos de ordenação por topologia e de caminho crítico. Utiliza-se o método de Merkel para modelar a altura de uma torre de resfriamento e poder avaliar o volume necessário de uma torre de resfriamento para cada rede de trocadores de calor. Um estudo de caso é utilizado para ilustrar cada passo a medida que a metodologia é desenvolvida, buscando prover fundamentos para um estágio conceitual durante o projeto de um sistema de água de resfriamento.

Palavras-chaves: Integração de processo, Sistema de água de resfriamento, Rede de trocadores de calor, Queda de pressão

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Chapter 1

Background

1.1 Introduction

Recirculating cooling water systems are widely used for waste heat disposal in different industrial processes. In these systems, waste process heat is mainly rejected by water evaporation in a cooling tower. Manufactures and process engineers have been required to design and operate their cooling water systems at high thermal performance, predicting some impacts caused by small deviations from design specifications (Cortinovis et al., 2009).

Some researchers have applied process integration techniques to increase the cooling water system performance in industry. Wang and Smith (1994) introduced a methodology based on pinch analysis to target the maximum cooling water reuse and to reduce the cooling water requirement through heat exchangers in series-parallel arrangement. Later, also applying pinch analysis, Kim and Smith (2001) studied a method to improve the cooling towers capacity in debottlenecking situations. Recent studies have used mathematical programming to achieve optimum designs of cooling water networks (Panjeshahi et al. (2009), Gololo and Majozi (2012)).

Conventional cooling water systems are designed with heat exchangers in parallel, demanding substantial amount of recirculating water. Cooling tower supplies fresh cooling water to every heat exchanger in parallel and, then, this water is recirculated to the tower, as can be seen in Figure 1.1a. Despite not requiring cooling water at this supply temperature, all heat exchangers are supplied with the same cooling water temperature in this case.

Cooling water reuse can come as a strategy to reduce water recirculation. Some heat exchangers may not require cooling water at fresh cooling water temperature and can operate properly at higher temperatures. In this case, their cooling water supply could come, partially or entirely, from other heat exchanger, as illustrated by Figure 1.1b.



(A) Network with no cooling water reuse



(B) Network with cooling water reuse

FIGURE 1.1: Different heat exchanger network arrangements

By reusing cooling water and saving recirculating water, the cooling tower may be debottlenecked in a retrofit project or be reduced in volume size during a grassroot design. However, since cooling water reuse leads to a series-parallel arrangement, the overall pressure drop for this type of network may become more complex to be evaluated. For a heat exchanger network in parallel or in series, the overall pressure drop can be simply determined by the maximum or the sum of the heat exchangers pressure drops, respectively. On the other hand, for a series-parallel arrangement, it is necessary additional tools to analyse the combination of both series and parallel layouts.

This study introduces a methodology to assess the impact of reusing cooling water on the heat exchanger network pressure drop and on the cooling tower size. Different heat exchanger networks are designed by using a superstructure model and the pressure drop for each one is computed by using Graph Theory algorithms. Cooling tower height is also designed to provide an analysis of the required volume for different arrangements. A case study is used as an illustration during the methodology development.

1.2 Aim and Objectives

The present study aims to propose and implement a methodology that designs different cooling water systems and that assesses the impact of reusing cooling water on the heat exchanger network pressure drop and on the cooling tower size.

The aim can be focused into the following objectives:

- To design different cooling water systems with cooling water reuse by:
 - i applying an algorithm that can model different cooling water system structures and that decomposes a Mixed-Integer Nonlinear programming (MINLP) problem into a Nonlinear programming (NLP) optimisation problem to achieve the minimum utility requirement.
- To propose a method to evaluate the pressure drop for heat exchanger networks by:

- i using the topological sort algorithm to detect cycles in a heat exchanger network;
- ii determining the critical path in order to evaluate the overall pressure drop in a heat exchanger network;
- To give conceptual insights of the cooling water reuse impacts on the different components of the cooling water system by:
 - i applying the methodology in a case study to illustrate some impacts of the cooling water reuse on the heat exchanger network pressure drop and on the cooling tower size;
 - analysing some impacts during an application of cooling water reuse for a grassroot and retrofit scenarios.

1.3 Methods

A mathematical programming method is applied in this study as a way of modelling different cooling water systems. A superstructure model is used in conjunction with a combinatorial algorithm to decompose a Mixed-Integer Nonlinear Programming (MINLP) problem into several Nonlinear Programming (NLP) problems. The decomposition is necessary to apply the Graph Theory algorithms and to evaluate the overall pressure drop in acyclic heat exchanger networks.

The algorithms are modelled by using Visual Basic for Applications (VBA) in Microsoft Excel 2013. Although other programming languages (Fortran, C, Pascal, etc) could be used as computational tool, VBA was selected because of its ubiquity in most computers in industry and its integration with the optimisation tool, Microsoft Excel Solver.

1.4 Thesis outline

Chapter 2 reviews Pinch Analysis, targeting the minimum cooling water requirement through a cooling water composite curve. Then, a superstructure model is applied to create different heat exchanger network arrangements. For each layout, Microsoft Excel Solver is used to minimise the utility requirement according to the system constraints. Finally, a case study illustrates the procedures that were described in this chapter.

Chapter 3 introduces a methodology to evaluate pressure drop in heat exchanger networks. Graph Theory concepts are used to represent the network structure. Then, the heat exchanger network pressure drop is evaluated by applying topological and critical path algorithms. The case study from Chapter 2 is also used in this chapter to explain how the procedure works.

Chapter 4 describes how to design some features of a cooling tower in a cooling water system. A quadratic curve is fitted into the water equilibrium curve to provide an analytical procedure that estimates the minimum required airflow. Then, the cooling tower height and performance are evaluated according to their operating conditions. The cooling tower volume requirement is also analysed for the case study described in the previous chapters.

Chapter 5 explores some impacts that the cooling water reuse can cause on a grassroot/retrofit scenarios. Some physical insights are analysed for different situations regarding heat transfer area and pumping system. Two algorithms are proposed to provide cooling water systems for the different scenarios.

Chapter 6 concludes the study providing some overviews and suggestions for future research.

Chapter 2

Heat exchanger networks

A cooling water system consists basically of a heat exchanger network, cooling towers and a pumping system (Figure 2.1) (Ponce-Ortega et al., 2010). Recirculating water is pumped from a cooling tower to a heat exchanger network, in which receives waste heat from a particular hot process. Cooling water returns to the tower to be cooled through direct contact with ambient air and, then, is recirculated into the system (Smith, 2005).



FIGURE 2.1: Cooling water system representation

The heat exchanger network contains different heat exchangers to transfer the waste heat from a particular hot process to the cooling water. A certain heat load must be removed from the hot process in order to reduce its temperature from inlet (T_{in}^{hot}) to outlet temperature (T_{out}^{hot}) . The desired heat transfer only happens if the cooling water is colder than the hot process stream, i.e., there is a temperature difference between the hot and cold streams to create a heat transfer driving force.

By considering ΔT_{\min} as the minimum temperature difference between the hot and cold streams, a limiting temperature profile can be created for each heat exchanger (Smith, 2005). A feasible region for the cooling water temperature (T^{CW}) in a countercurrent heat exchanger is illustrated in Figure 2.2. Both temperatures $T^{CW}_{in,max}$ and $T^{CW}_{out,max}$ limit the inlet and outlet cooling water temperatures to satisfy ΔT_{\min} , respectively (Kim and Smith, 2003).



(B) Limiting temperature profile for a heat exchanger

FIGURE 2.2: Heat exchanger temperature profile

The limiting temperature profiles for the different heat exchangers can be plotted together on the same graph, as can be seen in Figure 2.3a. By combining the profiles within each temperature interval, the separate streams can be represented by a single curve called *c*ooling water composite curve (Figure 2.3b) (Kemp, 2007).



FIGURE 2.3: Cooling water composite curve

Cooling water composite curve has the advantage of presenting the minimum cooling water requirement in a heat exchanger network. If water is supplied by a cooling tower at temperature $T_{\rm in}^{\rm net}$, a straight line can represent a cooling water supply line, as can be seen in Figure 2.4a. The line must be below the composite curve to satisfy the temperature profile constraints. Since its slope is inversely proportional to the cooling water flowrate, the maximum slope represents the minimum flowrate requirement (Figure 2.4b). As a disadvantage, some practical constraints cannot be determined by this curve, such as the heat exchangers arrangement and the pressure drop aspects (Kim and Smith, 2003).



FIGURE 2.4: Cooling water supply line curve

2.1 Cooling water reuse

Cooling water reuse is a strategy to reduce recirculating utility in a cooling water system. Differently from the composite curve analysis, this procedure takes into account the heat exchangers layout and their connecting streams.

In a parallel arrangement, the heat exchangers are totally supplied by fresh water from the cooling tower (Figure 2.5a). However, if the inlet temperature can be higher than the fresh cooling water temperature, some water from other cooler can be reused for a given heat exchanger (Figure 2.5b). As a result, the reuse stream leads the heat exchanger network to a series-parallel arrangement and its flowrate must be determined on the condition that the limiting temperature profile constraints are satisfied.



FIGURE 2.5: Cooling water reuse

By reusing cooling water, the total cooling water flowrate can be reduced down to the thermodynamic limit F_{\min} which is dictated by the composite curve (Figure 2.4b). In this study, water-saving efficiency (ε) is based on the definition given by Wang et al. (2013) (Equation 2.1) that receives the value 100% for an arrangement with maximum cooling water reuse and 0% for no cooling water reuse (parallel arrangement).

$$\varepsilon = \frac{F_{\min}^{\text{parallel}} - F}{F_{\min}^{\text{parallel}} - F_{\min}} \quad \forall \quad F_{\min} \le F \le F_{\min}^{\text{parallel}}$$
(2.1)

2.2 Superstructure model

A superstructure is a mathematical tool that can express all alternative streams for splitting, mixing and, in some cases, recycling and bypassing in a heat exchanger network (Kim and Smith, 2003). Every stream is associated to a binary variable $Y_{i,j}$ that defines if the stream exists ($Y_{i,j} = 1$) or not ($Y_{i,j} = 0$). The subscript of a variable $Y_{i,j}$ represents that a stream comes from the node *i* and goes to the node *j*.



FIGURE 2.6: General superstructure

A superstructure of two heat exchangers with some possible connecting streams is illustrated in Figure 2.6. In this model, the source node is represented by zero and the sink node by the number of heat exchangers plus one $(n_{\text{HE}} + 1)$. Defining the combination of $Y_{i,j}$, it is possible to make different arrangements, i.e., they can be arranged:

- in parallel layout if:
 - i $Y_{1,2}$ and $Y_{2,1}$ are zero and all the other $Y_{i,j}$ are one;
- in series layout if:
 - i $Y_{2,1}$, $Y_{1,3}$ and $Y_{0,2}$ are zero and all the other $Y_{i,j}$ are one, or;
 - ii $Y_{1,2}$, $Y_{2,3}$ and $Y_{0,1}$ are zero and all the other $Y_{i,j}$ are one;
- in series-parallel layout if:
 - i $Y_{1,2}$ is zero and all the other $Y_{i,j}$ are one, or;
 - ii $Y_{2,1}$ is zero and all the other $Y_{i,j}$ are one, or;
 - iii all $Y_{i,j}$ are one;
Cooling water reuse streams are represented by variables $Y_{i,j}$ whose both indexes *i* and *j* indicate different heat exchangers. As depicted in Figure 2.6, for example, both variables $Y_{1,2}$ and $Y_{2,1}$ represent cooling water reuse streams and, for this study, these variables receive a special superscript *reuse* $(Y_{i,j}^{\text{reuse}})$.

All variables $Y_{i,j}$ can be combined into a mathematical data structure which is called adjacency matrix. This matrix can be used to represent any heat exchanger network, expressing the connections among the heat exchangers, the source and sink nodes. The matrix elements value follows the same rule that is described for a superstructure model:

- $Y_{i,j} = 0$, if node *i* is not connected to node *j*;
- $Y_{i,j} = 1$, if node *i* is connected to node *j*.

The adjacency matrix structure of n_{HE} heat exchangers is illustrated in Figure 2.7. In this matrix, the source and sink nodes are expressed by the indexes zero and $n_{\text{HE}} + 1$, respectively. According to the elements values in this matrix, the heat exchangers can be arranged in parallel or series-parallel layouts, as described in the next sections.

$$Y = \begin{pmatrix} Y_{0,0} & Y_{0,1} & \cdots & Y_{0,n_{\rm HE}} & Y_{0,n_{\rm HE}+1} \\ Y_{1,0}^{\rm reuse} & Y_{1,1} & \cdots & Y_{1,n_{\rm HE}}^{\rm reuse} & Y_{1,n_{\rm HE}+1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ Y_{n_{\rm HE},0} & Y_{n_{\rm HE},1}^{\rm reuse} & \cdots & Y_{n_{\rm HE},n_{\rm HE}} & Y_{n_{\rm HE},n_{\rm HE}+1} \\ Y_{n_{\rm HE}+1,0}^{\rm reuse} & Y_{n_{\rm HE}+1,1} & \cdots & Y_{n_{\rm HE}+1,n_{\rm HE}}^{\rm reuse} & Y_{n_{\rm HE}+1,n_{\rm HE}+1} \end{pmatrix}$$

FIGURE 2.7: Adjacency matrix of a HEN

2.2.1 Parallel Arrangement

The parallel arrangement of a heat exchanger network is the most common layout for cooling water systems. In this arrangement, fresh cooling water is sent to the heat exchangers and no cooling water is reused. Its configuration in a superstructure model and respective adjacency matrix are illustrated in Figure 2.8.



FIGURE 2.8: Heat exchanger network in parallel arrangement

In the adjacency matrix for a parallel arrangement, the first row and last column are filled with ones, except for their first and last elements (Figure 2.8b). Since no cooling water is reused in this arrangement, the other elements receive the value zero.

2.2.2 Series-parallel arrangement

From a parallel configuration, the series-parallel arrangement can be created if any variable $Y_{i,j}^{\text{reuse}}$ from Figure 2.7 is one and, hence, there is, at least, one cooling water reuse among the heat exchangers. The streams that connect the heat exchangers to the source or sink nodes are fixed to ensure inlet and outlet streams for each heat exchanger.



(B) Adjacency matrix of a series-parallel arrangement



(C) Reuse stream array

The adjacency matrix for Figure 2.9a can be expressed by Figure 2.9b. By combining the variables Y^{reuse} into an array (Figure 2.9c), different series-parallel networks can be created from particular binary combinations.

The array Y^{reuse} size is calculated according to Equation 2.2. If the number of cooling water reuse streams (n_{reuse}) determines the quantity of ones in this array, the maximum number of different arrays is determined by the permutation of n_{reuse} ones in $n_{\text{reuse}}^{\text{max}}$ positions (Equation 2.3).

$$n_{\rm reuse}^{\rm max} = \frac{n_{\rm HE}!}{(n_{\rm HE} - 2)!}$$
 (2.2)

$$n_{\rm net}^{\rm max} = \frac{n_{\rm reuse}^{\rm max}!}{n_{\rm reuse}!(n_{\rm reuse}^{\rm max} - n_{\rm reuse})!}$$
(2.3)

2.3 Mass and energy balances

Mass and energy balances must be satisfied at each heat exchanger, mixing and splitting nodes, independently of the network arrangement. A heat exchanger *i* can be represented by Figure 2.10, whose heat load Q_i is removed from the hot process by a cooling water flow F_i at inlet temperature T_i^{in} (Kim and Smith, 2003).



FIGURE 2.10: Heat exchanger model

All cooling water flow rates can be arranged in a matrix F, in which each variable $F_{i,j}$ receives the cooling water flow rate value that comes from a node i and goes to a node j (Figure 2.11).

$$F = \begin{pmatrix} F_{0,0} & F_{0,1} & \cdots & F_{0,n_{\text{HE}}} & F_{0,n_{\text{HE}}+1} \\ F_{1,0}^{\text{reuse}} & F_{1,1} & \cdots & F_{1,n_{\text{HE}}}^{\text{reuse}} & F_{1,n_{\text{HE}}+1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ F_{n_{\text{HE}},0} & F_{n_{\text{HE}},1}^{\text{reuse}} & \cdots & F_{n_{\text{HE}},n_{\text{HE}}} & F_{n_{\text{HE}},n_{\text{HE}}+1} \\ F_{n_{\text{HE}}+1,0}^{\text{reuse}} & F_{n_{\text{HE}}+1,1} & \cdots & F_{n_{\text{HE}}+1,n_{\text{HE}}}^{\text{reuse}} & F_{n_{\text{HE}}+1,n_{\text{HE}}+1} \end{pmatrix}$$

FIGURE 2.11: Cooling water flowrate matrix

By combining the matrix F with the adjacency matrix Y, the inlet and outlet cooling water flowrates of a heat exchanger i can be defined by Equations 2.4 and 2.5, respectively.

$$F_i^{\rm in} = \sum_{j=0}^{n_{\rm HE}+1} Y_{j,i} \times F_{j,i} \quad \forall \quad 1 < i \le n_{\rm HE}$$
(2.4)

$$F_i^{\text{out}} = \sum_{j=0}^{n_{\text{HE}}+1} Y_{i,j} \times F_{i,j} \quad \forall \quad 1 < i \le n_{\text{HE}}$$

$$(2.5)$$

Since both Equations 2.4 and 2.5 must be equal to satisfy the mass balance, the following constraint must be satisfied:

$$F_i^{\rm in} - F_i^{\rm out} = 0 \tag{2.6}$$

By applying Equations 2.4 and 2.5 in the source (i = 0) or in the sink $(i = n_{\text{HE}} + 1)$ node, the total cooling water flowrate for a given heat exchanger network can be calculated by Equation 2.7.

$$F_{\text{net}}^{\text{total}} = \sum_{j=0}^{n_{\text{HE}}+1} Y_{i,j} \times F_{i,j} \quad \forall \quad i = 0 \quad \text{or} \quad i = n_{\text{HE}} + 1$$
(2.7)

Assuming C_P is constant and T_{in}^{net} as the inlet network temperature, the inlet (T_i^{in}) and outlet (T_i^{out}) temperatures for a given heat exchanger *i* are calculated by Equations 2.8 and 2.9, respectively.

$$T_{i}^{\rm in} = \frac{Y_{0,i}F_{0,i}T_{\rm in}^{\rm net} + \sum_{j=1}^{n_{\rm HE}+1}Y_{j,i}F_{j,i}T_{j}^{\rm out}}{\sum_{j=0}^{n_{\rm HE}+1}Y_{j,i}F_{j,i}}$$
(2.8)

$$T_{i}^{\text{out}} = T_{i}^{\text{in}} + \frac{Q_{i}}{C_{P} \sum_{j=0}^{n_{\text{HE}}+1} Y_{j,i} F_{j,i}}$$
(2.9)

A closed loop occurs if Equations 2.8 and 2.9 are combined, since the inlet temperature (T_i^{in}) equation depends on the outlet temperature (T_i^{out}) and vice versa. This circular reference can be eliminated if a new variable $T_i^{\text{out},*}$ is created to substitute T_i^{out} in Equation 2.8. In this approach, the variable $T_i^{\text{out},*}$ is calculated on the condition that the following constraint is satisfied.

$$T_i^{\text{out}} - T_i^{\text{out},*} = 0 \tag{2.10}$$

The limiting temperature profile provides the maximum inlet and outlet temperatures to give ΔT_{\min} throughout the heat exchangers. From the limiting temperatures, the inlet and outlet temperatures of a heat exchanger *i* must satisfy the following inequalities.

$$T_i^{\rm in} - T_i^{\rm in,max} \le 0 \tag{2.11}$$

$$T_i^{\text{out}} - T_i^{\text{out,max}} \le 0 \tag{2.12}$$

Following the same principle that was explained for cooling water composite curve, the minimum cooling water requirement for a single heat exchanger can also be depicted by a temperature versus enthalpy graph. Considering $T_{\text{in}}^{\text{net}}$ is the minimum cooling water temperature that can be supplied in a heat exchanger *i*, the minimum cooling water flowrate can be calculated by the maximum slope (α) for the cooling water line (Figure 2.12).



FIGURE 2.12: Minimum cooling water flowrate for a heat exchanger i

Therefore, the minimum cooling water flowrate for a heat exchanger i can be calculated by Equation 2.13.

$$F_i^{\min} = \frac{Q_i}{C_P(T_i^{\text{out,max}} - T_{\text{in}}^{\text{net}})}$$
(2.13)

By computing the minimum cooling water requirement (F_i^{\min}) for each heat exchanger, the following constraint must also be satisfied (Equation 2.14).

$$F_i^{\min} - \sum_{j=0}^{n_{\rm HE}+1} Y_{j,i} F_{j,i} \le 0$$
(2.14)

2.4 Cooling water flowrate minimisation

Mass and energy balances can be implemented into a mixed-integer nonlinear programming (MINLP) problem optimisation to obtain the arrangement that requires the minimum cooling water flowrate. The objective function to determine the minimum cooling water requirement can be defined by Equation 2.15.

$$F_{\text{net}}^{\min} = \min\left(\sum_{j=0}^{n_{\text{HE}}+1} Y_{0,j} \times F_{0,j}\right)$$
(2.15)

This function can be minimised on the condition that every constraint from the previous section is satisfied. The decision variables $Y_{i,j}$, $F_{i,j}$ and $T_i^{\text{out},*}$ can be adjusted not only to satisfy the mass and energy balances constraints, but also to give the arrangement with minimum cooling water flowrate. The initial values for $F_{i,j}$ and $T_i^{\text{out},*}$, are set to be the maximum outlet temperature $T_i^{\text{out},\text{max}}$ and the minimum flowrate F_i^{min} , respectively. The other variables $F_{i,j}$ receive the value zero and are adjusted during the minimisation.

As a limitation, different solutions may be required during a conceptual design and the single mathematical solution that is given by a MINLP problem may not suit engineering design aspects and/or not give other possible design option. In this approach, the study has focused on modelling different heat exchanger networks by modifying the binary variables $Y_{i,j}$ and solving each nonlinear problem for a given adjacency matrix.

Therefore, in this study, different networks are created by modifying the adjacency matrix and Microsoft Excel Solver is used to minimise the nonlinear programming (NLP) problem for each heat exchanger structure. Heap's algorithm is applied to make all different combinations in the array Y (Figure 2.9c) for a given reuse streams number (n_{reuse}) . This algorithm generates recursively all possible permutations of a number of objects and can be used to permute the number of ones (n_{reuse}) and zeroes $(n_{\text{reuse}}^{\text{max}} - n_{\text{reuse}})$ in the array Y^{reuse} (Heap, 1963). Algorithm 1 Heap's Algorithm

```
procedure HEAP(n_1, n_0, i, j, temp)
   if n_1 = 0 then
       for k = 1 to n_0 Step 1 do
           Y(i)(j) \leftarrow 0
          i + +
       end for
       i + +
       return
   else if n_0 = 0 then
       for k = 1 to n_1 Step 1 do
          Y(i)(j) \leftarrow 1
          j + +
       end for
       i + +
       return
   end if
   Y(i)(j) \leftarrow 1
   for k = 1 to j Step 1 do
       temp(k) = Y(i)(j)
   end for
   HEAP(n_1 - 1, n_0, i, j + 1, temp())
   for k = 1 to j - 1 Step 1 do
       Y(i)(j) = temp(k)
   end for
   Y(i)(j) \leftarrow 0
   HEAP(n_1, n_0 - 1, i, j + 1, temp())
end procedure
```

The algorithm that combines the nonlinear programming minimisation and the Heap's algorithm is illustrated in Figure 2.13. Varying the adjacency matrix by the Heap's algorithm,

the procedure minimises the cooling water requirement for different arrangements.



FIGURE 2.13: Algorithm to model different heat exchanger networks

2.5 Case study application

This section applies the methodology for a heat exchanger network whose limiting cooling water data was adapted from Smith (2005). The limiting temperature profiles for four different operations and their respective heat load Q_i are expressed in Table 2.1.

Heat exchanger	$T_i^{\text{in,max}}$ (°C)	$T_i^{\text{out,max}}$ (°C)	$Q_i \; (\mathrm{kW})$
1	20	40	400
2	30	40	1000
3	30	75	1800
4	55	75	200

TABLE 2.1: Limiting cooling water data (Adapted from Smith (2005)).

The composite curve can be plotted according to the limiting temperatures and the heat load for each heat exchanger. Considering that a cooling tower supplies fresh cooling water at 20 °C, the minimum cooling water flowrate can be obtained graphically by the composite curve, as illustrated in Figure 2.14.



FIGURE 2.14: Composite curve

By arranging the heat exchangers in a parallel arrangement, no cooling water is reused in the network and the minimum cooling water flowrate is obtained if each heat exchanger is supplied by its respective minimum flowrate F_i^{\min} (Equation 2.13). For this layout, the cooling water requirement is 25.5 kg s^{-1} , as illustrated in Figure 2.15. If this value is compared to the minimum cooling water flowrate dictated by the composite curve (Figure 2.14), it is possible to verify that up to 4.0 kg s^{-1} can be reduced by reusing cooling water.



FIGURE 2.15: Cooling water flowrate for the parallel arrangement - water-saving efficiency ε of 0% (F in kg s⁻¹ and Q in kW)

A cooling water reuse stream can be created if, at least, one variable Y^{reuse} is different from zero in the adjacency matrix (Figure 2.9b). By permuting and increasing the amount of ones in this matrix, it is possible to create a large number of different heat exchanger networks. However, depending on Y^{reuse} combination, some reuse streams may not be useful to reduce recirculating cooling water and, this way, the structure may remain unchanged (i.e., in parallel).

In the present case study, the number of reuse streams (n_{reuse}) indicates the number of ones in the array Y^{reuse} . For $n_{\text{reuse}} = 1$, twelve series-parallel arrangements can be created as indicated by Equation 2.3. However, after minimisation, only four networks converge to a lower cooling water requirement than for the parallel structure. The other eight heat exchanger networks remain equivalent to the parallel arrangement, since their respective cooling water reuse streams cannot contribute to reduce recirculating cooling water.

For both arrangements 1 and 2 (Figure 2.16), 1.4 kg s^{-1} of cooling water is reused in the heat exchanger 4, resulting in a water-saving efficiency ε of 22.2%. For the arrangements 3

and 4 (Figure 2.17), 3.1 kg s^{-1} of cooling water is saved by reusing recirculating water in the heat exchanger 3, equivalent to a water-saving efficiency ε of 77.8% (Equation 2.1). As can be noticed, this value is the maximum water-saving efficiency that can be achieved for one single reuse. Therefore, in order to achieve the minimum cooling water flowrate dictated by the composite curve, more than one reuse stream is necessary.



FIGURE 2.16: Arrangements with one reuse stream and water-saving efficiency ε of 22.2% (*F* in kg s⁻¹ and *Q* in kW)



FIGURE 2.17: Arrangements with one reuse stream and water-saving efficiency ε of 77.8% (F in kg s⁻¹ and Q in kW)

For $n_{\rm reuse} = 2$, it is possible to create 66 different heat exchanger networks, as can be

calculated by Equation 2.3. However, among them, only seven combinations converge to structures that contain two reuse streams. The other 59 networks are equivalent to the previous arrangements, as shown in Table 2.2.

Reuse Stream	Number of networks	%
0	28	42.4
1	31	47.0
2	7	10.6
Total	66	100

TABLE 2.2: Networks number distribution

The seven different networks with two reuse streams are presented in Appendix A. Among them, the arrangements that achieved the highest water-saving efficiency are illustrated in Figure 2.18.



(C) Arrangement 3

FIGURE 2.18: Arrangements with two reuse stream and water-saving efficiency ε of 100% (F in kg s⁻¹ and Q in kW)

As can be seen in Figure 2.18, three arrangements converged to the maximum water-saving efficiency ε (100%). Since two reuse streams are sufficient to achieve the maximum limit of water-saving efficiency ε (100%), the increase of the number n_{reuse} becomes unnecessary since this strategy may result in complex and impractical structures.

Chapter 3

Pressure drop in cooling water network

Pressure drop is an important issue to take into account during the heat exchanger network design. The pump system needs to provide enough energy to overcome pressure losses due to cooling water flow through heat exchangers. Its associated cost may represent a significant part of the overall expenditure to build and operate a cooling water system.

Kim and Smith (2003) introduced a linear-programming (LP) to evaluate the pressure drop in a heat exchanger network. The network is represented by a superstructure model, in which each mixing or splitting node i contains a pressure value P_i , as can be seen in Figure 3.1. On the condition that the constraint in Equation 3.2 is satisfied, the pressure drop in the network is obtained by minimising Equation 3.1.



FIGURE 3.1: Superstructure model for pressure drop analysis

$$\Delta P^{\rm net} = \min\left(P_{\rm source} - P_{\rm sink}\right) \tag{3.1}$$

$$P_i - P_j \ge \Delta P_{ij} \quad \forall \quad i \neq j \tag{3.2}$$

As a limitation, only the overall pressure drop calculated by Equation 3.1 is meaningful in this methodology. Besides procedure cannot provide the critical heat exchanger that influences the overall pressure drop in the network, no convergence can be obtained for cyclic networks.

Cyclic heat exchanger networks are formed if the connecting streams creates a directed cycle among the heat exchangers. A cycle consists of a sequence of vertices that start and end at the same vertex. An example of a simple cycle between two heat exchangers is shown in Figure 3.2.



FIGURE 3.2: Example of a cycle in a network

The following sections present a methodology to evaluate the overall pressure drop in a heat exchanger network based on the Graph Theory. First, a correlation is used to compute the pressure drop for the cooling water side in each heat exchanger. Then, Graph Theory algorithms are applied to detect cycles and to evaluate the critical path in an acyclic network.

3.1 Pressure drop correlation

The pressure drop for the cooling water side can be estimated by different methodologies in the literature. However, for an initial analysis during a heat exchanger network synthesis, it is convenient to choose a correlation that requires little information about the heat exchanger structure (Smith, 2005). In this study, the model introduced by Smith (2005) is used, since it depends very little on the detailed heat exchanger geometry. The correlation relates the pressure drop to the heat transfer coefficient and the heat exchanger surface area, as shown in Equation 3.3.

$$\Delta P_{\rm T} = K_{\rm PT1} A h_{\rm T}^{3.5} + K_{\rm PT2} h_{\rm T}^{2.5} \tag{3.3}$$

In which:

$$K_{\rm PT1} = \frac{0.023\rho^{0.8}\mu^{0.2}d_{\rm i}^{0.8}}{V_{\rm i}d_{\rm o}} \left(\frac{1}{K_{\rm hT}}\right)^{3.5}$$
(3.4)

$$K_{PT2} = 1.25 N_{\rm TP} \rho \left(\frac{1}{K_{\rm hT}}\right)^{2.5}$$
 (3.5)

$$h_{\rm T} = K_{\rm hT} v_{\rm T}^{0.8} \tag{3.6}$$

$$K_{\rm hT} = 0.023 \left(\frac{k}{d_{\rm i}}\right) P r^{\frac{1}{3}} \left(\frac{d_{\rm i}\rho}{\mu}\right)^{0.8} \tag{3.7}$$

For a single-pass countercurrent heat exchanger, the heat transfer area can be calculated by Equation 3.8.

$$A_i = \frac{Q_i}{U_i \Delta T_{\rm lm}} \tag{3.8}$$

In which:

$$\Delta T_{\rm lm} = \frac{(T^{\rm out,hot} - T_i^{\rm in}) - (T_i^{\rm in,hot} - T_i^{\rm out})}{\ln\left(\frac{(T^{\rm out,hot} - T_i^{\rm in}))}{(T_i^{\rm in,hot} - T_i^{\rm out})}\right)}$$
(3.9)

$$\frac{1}{U} = \frac{1}{h_{\rm S}} + R_{\rm S}^{\rm f} + \frac{d_{\rm o}}{2k} \ln\left(\frac{d_{\rm o}}{d_{\rm i}}\right) + \frac{d_{\rm o}}{d_{\rm i}}\frac{1}{h_{\rm T}} + \frac{d_{\rm o}}{d_{\rm i}}R_{\rm T}^{\rm f}$$
(3.10)

According to Müller-Steinhagen (2010), cooling water is typically used in the tube side of shell and tubes heat exchangers at velocities about 1 m s^{-1} and 2 m s^{-1} . Furthermore, the fouling resistance can be estimated according to the cooling water bulk temperature. For cooling water at high temperatures, the fouling resistance may increase because of the inverse solubility of some salts in water, such as CaCO₃, CaSO₄, Ca₃(PO₄)₂, CaSiO₃, Ca(OH)₂, $Mg(OH)_2$, $MgSiO_3$, Na_2SO_4 , Li_2SO_4 , and Li_2CO_3 . In this context, for velocity of 1 m s^{-1} , if the bulk temperature is less or equal to 50 °C, the fouling resistance for the water is around $0.53 \text{ m}^2 \text{ K kW}^{-1}$, for over 50 °C, it may increase to $0.7 \text{ m}^2 \text{ K kW}^{-1}$ (Müller-Steinhagen, 2010).

3.2 Pressure drop in a heat exchanger network

After evaluating the pressure drop for each single heat exchanger, the overall pressure drop in a heat exchanger network can be estimated if the network layout is well-known (Kim and Smith, 2003). There are only two ways to arrange two heat exchangers: in series (Figure 3.3a) or in parallel (Figure 3.3b). If the units are connected in series, the pressure drop is calculated as the sum of the pressure drop in the two heat exchangers. For a parallel arrangement, the pressure drop is equivalent to the maximum value of the two. For more than two heat exchangers, it is possible to create a series-parallel layout, whose pressure drop is evaluated by combining both series and parallel properties (Figure 3.3c).



FIGURE 3.3: Heat exchangers arrangement types

3.2.1 Graph representation

Any structure of a heat exchanger network can be represented by a graph — a very versatile model that can be used to analyse a wide range of practical problems. They consist of circles (or dots) and connections which can have some physical or conceptual interpretations (Gross and Yellen, 2005). If the connection is directed (its direction is indicated by an arrow), the graph is called directed graph or digraph. Mathematically, the digraph can be

represented by an adjacency matrix, in which the entry $a_{i,j} = 1$ if there is an arrow from vertex *i* to vertex *j* and $a_{i,j} = 0$ if otherwise.



FIGURE 3.4: Digraph representation

In order to evaluate the pressure drop, the heat exchanger network can be represented by a digraph, in which the circles and arrows symbolise, respectively, pressure drop points and flow directions in the network. If pipe pressure drops are neglected, the circles can be simplified to only represent heat exchanger pressure drops (P_i^{drop}) .

3.2.2 Topological Sorting Algorithm

The topological sorting algorithm is useful for defining the order the pressure drop must be evaluated in a heat exchanger network. Assuming the vertices and arrows represent the heat exchangers and connecting streams respectively, the algorithm gives the sequence that is required to evaluate the overall pressure drop.

As a limitation, this algorithm can only be used for digraphs with no cycles, also known as directed acyclic graphs or simple acyclic digraphs. To create a cycle in a heat exchanger network, a recycling pump is required in the network to recycle part of the cooling water among the heat exchangers. By inserting this pump, the water can be pumped from a low to a high pressure point. The topological sorting algorithm used in this study is based on depth-first search (Algorithm 2). This algorithm is a recursive function that can detect the existence of cycles and provide the topological sort if the graph is acyclic.

```
Algorithm 2 Topological Sorting Algorithm - Part 1
```

```
Require: A()(), n_{\text{HE}}
Ensure: toporder()
  m(n_{\rm HE}+2) \leftarrow 0
  k \leftarrow n_{\rm HE} + 2
  function (cycle)(A()(), n_{HE})
      for i = 1 to n_{\text{HE}} + 2 do
          if m(i) = 0 then
              if visit(A()(), i, n_{HE}, k, toporder())=1 then
                 cycle = 1
                 return
              end if
          end if
      end for
  end function
  function (visit)(A()(), i, n_{\text{HE}}, k, toporder())
      m(i) = 1
      for j = 1 to n_{\text{HE}} + 2 do
          if A(i)(j) = 1 then
              if m(j) = 1 then
                 visit = 1
                 return
              else if m(j)=0 then
                 if visit(A()(), i, n_{HE}, k, toporder())=1 then
                     visit = 1
                     return
                 end if
              end if
          end if
      end for
      m(i) = 2
      toporder(k) = i
      k - -
      visit = 0
  end function
```

3.2.3 Critical Path Algorithm

The critical path (or longest path) algorithm is commonly applied for scheduling a set of project activities (PM, 2013). In this context, the algorithm calculates the longest path of planned activities, determining the shortest time possible to complete a project. Furthermore, it indicates the activities which are "critical" (i.e., makes the project longer if delayed) and "total float" (i.e., does not make the project longer if delayed) (Sears, 2008).

The tasks durations follow the same principle described for pressure drop in the beginning of Section 3.2. If two tasks can be performed at the same time (i.e., in parallel), the required time to accomplish both tasks is the longest task duration. In case a task must be done before other (i.e., in series), it is required the summation of the tasks duration to complete both ones.

For project activities, the algorithm starts calculating the earliest start time for each task according to Equation 3.11 (Zhao and Tseng, 2003). This equation indicates that the earliest start time ES of an activity j is the maximum value of its predecessors ES_i added to its respectively duration time D_i .

$$\mathrm{ES}_{j} = \max\left\{\mathrm{ES}_{i} + D_{i} \| \ i \in P_{j}\right\} \quad \text{for}$$

$$(3.11)$$

By assigning the zero start value for the first activity, the earliest start time values are calculated successively. As soon as the last activity is calculated, the latest start time (LS) variable is created to receive the maximum value from the earliest start time (ES) variables. Then, a backward pass method is done following Equation 3.12. This equation indicates that the latest start time value of a predecessor i is equal to the minimum value of its successors LS_i minus their respective duration time D_i (Zhao and Tseng, 2003).

$$LS_i = \min \left\{ LS_j - D_i \| \ j \in S_i \right\}$$

$$(3.12)$$

After calculating the ES and LS for every activity, the critical (or longest) path is determined as the path which contains activities with the same value for ES and LS (Vukmirovic et al., 2012). Furthermore, the project duration corresponds to their maximum value, i.e., the final activity value for either ES or LS.

3.2.4 Critical path application

Besides management scheduling applications, the critical path algorithm can also be used for determining the pressure drop in heat exchanger networks. In this case, instead of managing an activity duration (i.e. time), the critical path calculation deals with the heat exchanger pressure drop (i.e. pressure). The analogy between the variables for the two different applications can be illustrated in Figure 3.5.



First, the array P_i^{drop} that contains the pressure drop for each heat exchanger is used rather than the duration array D_i . Second, the variables ES and LS can be replaced by $P_{\text{in},i}^{\min}$ and $P_{\text{in},i}^{\max}$, representing the minimum and maximum inlet pressure for a heat exchanger *i*, respectively. The difference between them represents a slack pressure that a heat exchanger can receive in its inlet. If both values are equal, the heat exchanger i has a fixed inlet pressure and is critical for the whole network.

From an adjacency matrix, both topological and critical path algorithms must be applied to evaluate the overall pressure drop in a heat exchanger network. It is important to note that, in an adjacency matrix, the row and column indexes represent the predecessor and successor vertices, respectively. Thus, the entry $a_{i,j} = 1$ means that there is a connection from the vertex *i* (predecessor) to the vertex *j* (successor). This definition is very important during the critical path algorithm, since the minimum inlet pressure and maximum inlet pressure calculation involves the relationship between predecessor and successor vertices.

During the pressure drop evaluation, a successor vertex must be only considered after every predecessor vertices. To follow this sequence, the adjacency matrix A and the array $P_{\rm drop}$ can be sorted to follow the topological order, as depicted in Figure 3.6.



FIGURE 3.6: Arrangement of the adjacency matrix A and the array P_{drop}

After arranging both rows and columns, the sorted adjacency matrix A^* and array P^*_{drop} are applied in the critical path algorithm to determine the critical path and the heat exchanger network pressure drop, as described by Algorithm 3.

To apply the critical path algorithm, first, two arrays of n vertices are created, the $P_{\text{in}}^{\text{max}}(n)$ and the $P_{\text{in}}^{\min}(n)$. If a pump is considered at the source node, the initial value for $P_{\text{in}}^{\max}(i)$ must be equivalent to the pressure that the pump can deliver to the network. This procedure must be taken since the pump pressure is the maximum possible pressure in every heat exchanger before considering the pressure drops.

The algorithm starts from the first row of the sorted matrix A^* until the last one, calculating the maximum inlet pressure for each vertex j ($P_{\text{in}}^{\text{max}}(j)$) according to Equation 3.13.

$$P_{\rm in}^{\rm max}(j) = \min \left\{ P_{\rm in}^{\rm max}(i) - a(i,j) \times P_{\rm drop}^{*}(i) \right\}$$
(3.13)

As the last row is evaluated, the $P_{\text{in}}^{\min}(n)$ receives the $P_{\text{in}}^{\max}(n)$ value and the minimum inlet pressure $(P_{\text{in}}^{\min}(i))$ is evaluated by a backward pass method, according to Equation 3.14.

$$P_{\rm in}^{\rm min}(i) = \max\left\{P_{\rm in}^{\rm min}(j) + a(i,j) \times P_{\rm drop}^{*}(i)\right\}$$
(3.14)

Finally, the critical path can be determined by the vertices whose $P_{\text{in}}^{\text{max}}(i)$ and $P_{\text{in}}^{\text{min}}(i)$ values are equal. Furthermore, the pressure drop of the heat exchanger network corresponds to the difference between the pressures in the source and sink nodes.

Algorithm 3 Critical Path Algorithm

```
Require: A^*()(), Pdrop^*(), n_{\text{HE}}
Ensure: Pmin(), Pmax(), DP
```

procedure (criticalpath) $(A^*()(), Pdrop^*()(), n_{\text{HE}})$

```
for i = 1 to n_{\text{HE}} + 2 Step 1 do
       if Pmin(i) < Pmin(j) + A^*(i)(j) * Pdrop(i) then
           Pmin(i) = Pmin(j) + A^{*}(i)(j) * Pdrop(i)
       end if
   end for
   for i = 1 to n_{\text{HE}} + 2 do
       Pmax(i) = Pmin(n_{\rm HE} + 2)
   end for
   for i = n_{\text{HE}} + 2 to 1 Step -1 do
       if Pmax(i) > Pmax(i) - A^*(i)(j) * Pdrop(i) then
           Pmax(i) = Pmax(i) - A^{*}(i)(j) * Pdrop(i)
       end if
   end for
   for i = 1 to n_{\text{HE}} + 2 do
       Critical if Pmin(i) = Pmax(i)
   end for
   DP = Pmin(1) - Pmin(n_{\rm HE} + 1)
end procedure
```

3.3 Case study application

Pressure drop evaluation is applied in the same case that was studied in the previous chapter. After defining the cooling water flowrate for a specific network, pressure drop in each heat exchanger can be estimated by the correlation from Section 3.1. For this correlation, some assumptions are required, as follow:

- Shell-and-tube heat exchangers with single pass (1-1);
- Cooling water stream flows in the tubes in counter-current with the hot process stream (in the shell side);

- Pipe pressure drops are considered negligible compared to the heat exchangers pressure drops;
- Cooling water stream velocity in tubes is 1 m s^{-1} ;
- Tubes outside diameter is 3/4 inch;
- Tubes thickness is 2×10^{-3} m;
- Heat transfer coefficient for the shell side $(h_{\rm S})$ is 800 W °C⁻¹ m⁻²;
- ΔT_{\min} is 20 °C;
- Cooling water properties are constant (25 °C): $\rho = 997 \text{ kg m}^{-1}$, $\mu = 0.890 \text{ }11 \times 10^{-3} \text{ Pa s}$, $k = 0.607 \text{ }15 \text{ W m}^{-1} \text{ K}^{-1}$, $C_P = 4181.6 \text{ J kg}^{-1} \text{ K}^{-1}$;
- Fouling resistance for the tube side is $0.53 \times 10^{-3} \text{ m}^2 \text{ KW}^{-1}$ for $T_i^{out} \leq 50 \text{ °C}$ and $0.7 \times 10^{-3} \text{ m}^2 \text{ KW}^{-1}$ for $T_i^{out} > 50 \text{ °C}$ (Müller-Steinhagen, 2010);
- Conduction resistance is negligible;
- Fouling resistance for the shell side $(R_{\rm S}^{\rm f})$ is negligible.

By applying the topological algorithm, the acyclic networks number can be computed for different numbers of reuse streams (n_{reuse}). For more than one reuse stream, the acyclic network condition reduces the number of series-parallel arrangements, as depicted in Figure 3.7. As can be seen in Table 3.1, there are 4,096 different networks that can be created with four heat exchangers, but, in fact, this number is reduced to 746 networks by computing only acyclic networks.

$n_{\rm reuse}$	$n_{\rm net}^{\rm max}$	$n_{\rm net,acyclic}^{\rm max}$	Cumulative $n_{\rm net}^{\rm max}$	Cumulative $n_{\rm net,acyclic}^{\rm max}$
0	1	1	1	1
1	12	12	13	13
2	66	60	79	73
3	220	156	299	229
4	495	222	794	451
5	792	181	1586	632
6	924	87	2510	719
7	792	24	3302	743
8	495	3	3797	746
9	220	0	4017	746
10	66	0	4083	746
11	12	0	4095	746
12	1	0	4096	746

TABLE 3.1: Number of different networks with cooling water reuse for $n_{\rm HE} = 4$



FIGURE 3.7: Series-parallel network possibilities as function of the number of cooling water reuse streams in a case of four heat exchangers

For a parallel arrangement, the critical path is simple calculated by the maximum pressure drop among the heat exchanger, as can be seen in Figure 3.8. The overall pressure drop for this heat exchanger network is equivalent to the pressure drop in the heat exchanger 3 (i.e., 62.1 kPa). For networks with cooling water reuse streams, the topological and critical path algorithms are applied to evaluate their respective critical paths and overall pressure drops.



FIGURE 3.8: Pressure drop in a parallel arrangement

Assuming the heat exchanger networks with one reuse stream that were described in the previous chapter, their critical paths are illustrated in Figures 3.9 and 3.10. For the arrangements whose water-saving efficiency is 22.2 %, the overall pressure drop remains close to the parallel network, around 62 kPa. However, for the arrangements with 77.8% of water-saving efficiency, the overall pressure drop increases 40%, on average.



FIGURE 3.9: Pressure drop in heat exchanger networks with one reuse stream and watersaving efficiency ε of 22.5 % (P in kPa and Q in kW)



FIGURE 3.10: Pressure drop in heat exchanger networks with one reuse stream and watersaving efficiency ε of 77.8 % (P in kPa and Q in kW)
For two reuse streams, it is only presented in this section the critical path for the three acyclic networks that achieve the maximum water-saving efficiency ($\varepsilon = 100\%$) (Figure 3.11). The other arrangements with two reuse streams and their respective critical path can be found in Appendix B. The results for each heat exchanger network can be summarised in Table 3.2. As can be seen for the arrangements with one reuse stream, a similar effect occurs for two reuse streams. If water-saving efficiency is 22.2 %, the overall pressure drop remains very similar to the parallel layout. However, for arrangements whose water-saving efficiency is above 22.2 %, there are more than one critical heat exchangers and the overall pressure drop increases in about 40%.

The hydraulic power that is required to pump the cooling water into the network can be estimated by Equation 3.15. As can be seen in Table 3.2, although the cooling water recirculation can be reduced up to $4.0 \,\mathrm{kg \, s^{-1}}$, an increase of about 15% in the hydraulic power may be required. A more detailed technical-economic analysis must be done in this case, since this increase may demand more electric power to pump the cooling water in the network, thereby increasing some operational expenditures.

$$W_{\rm h} = \frac{\Delta P_{\rm net} \ F}{\rho} \tag{3.15}$$

ε (%)	$F (\rm kg s^{-1})$	$\Delta P_{\rm net}^*$ (kPa)	$W_{\rm h}~({\rm kW})$
0.0	25.5	62.1	1.59
22.2	24.6	63.0	1.55
59.3	23.1	96.9	2.25
77.8	22.4	88.5	1.98
79.6	22.3	87.7	1.96
100.0	21.5	84.4	1.82

TABLE 3.2: Hydraulic power behaviour for different heat exchanger networks

* average



(C) Arrangement 3

FIGURE 3.11: Pressure drop in heat exchanger networks with two reuse stream and watersaving efficiency ε of 100.0% (P in kPa and Q in kW)

Chapter 4

Cooling towers and the cooling water network

Cooling tower is a heat exchanger that uses direct contact between ambient air and hot water in order to reduce the cooling water temperature. The heat is mostly rejected by water evaporation to the atmosphere, cooling the hot water up to wet-bulb temperature of the ambient air (T_{wet}) .

The classification of cooling towers is normally based on the type of draft: mechanical draft (forced convection) and natural draft (natural convection). On the one hand, the mechanical draft tower has a fan to draw air into the tower in counter or crosscurrent flow. The natural draft, on the other hand, relies on the buoyancy effect of the heated air that rises naturally due to the lower density if compared to the dry and cool outside air.

A counterflow mechanical draft tower integrated with a cooling water system is illustrated in Figure 4.1. The hot water that comes from the heat exchanger network flows downward through the packing and is cooled mainly by evaporation. Water vapour and drift leave the top of the tower with the humid and heated airflow. A blowdown current is necessary to prevent the contaminants accumulation in the recirculating water. Makeup water is added into the system to compensate the water losses from evaporation, drift and blowdown. Then, the



FIGURE 4.1: Recirculating cooling water scheme

fresh cooling water is pumped to the heat exchangers, in which the waste heat is transferred from the hot process to the cooling water (Smith, 2005).

4.1 Cooling tower model

The traditional procedure to design a counter-current cooling tower is based on the method developed by Merkel and Verdunstungskühlung (1925). The method evaluates the cooling tower height z_{tower} by Equation 4.1, considering the following assumptions:

$$z_{\text{tower}} = \frac{L C_{P,L}}{K_x a} \int_{T_{L,\text{out}}}^{T_{L,\text{in}}} \frac{\mathrm{d}T}{H_{G}^{\text{sat}} - H_{G,\text{op}}(T_L)}$$
(4.1)

- Lewis number of unity;
- Low mass transfer rate theory is valid;
- The liquid-side heat-transfer resistance is negligible;

- The amount of water evaporated is small and the water and air flowrates are constant;
- Adiabatic operation;
- Drift and leakage losses are neglected;

The enthalpy of the saturated air at the water-air interface $H_{G,in}$ can be calculated for a given air condition from the correlations taken from ASHRAE (1993). The water vapour pressure for the temperature range of 0 to 200 °C can be calculated by an adjustment equation, described by Equation 4.2.

$$\ln P^{\text{sat}} = \frac{C_1}{T} + C_2 + C_3 T + C_4 T^2 + C_5 T^3 + C_6 \ln T \quad T \text{ in K and } P^{\text{sat}} \text{ in Pa}$$
(4.2)

In which:

- $C_1 = -5.800\,220\,6 \times 10^3$
 - $C_2 = 1.3914993$
- $C_3 = -4.8640239 \times 10^{-2}$
- $C_4 = 4.176\,476\,8 \times 10^{-5}$
- $C_5 = -1.445\,209\,3 \times 10^{-8}$
- $C_6 = 6.5459673 \times 10^{-8}$

For a given atmospheric pressure P^{atm} , the humidity ratio W is calculated according to Equation 4.3.

$$W = \frac{M_{\text{water}}}{M_{\text{dry}}} \frac{P^{\text{sat}}}{P^{\text{atm}} - P^{\text{sat}}}$$
(4.3)

In which:

 $M_{\rm water} = 18.015\,{\rm kg\,kmol^{-1}}$ $M_{\rm dry} = 28.966\,{\rm kg\,kmol^{-1}}$

For a given wet-bulb temperature, the enthalpy is determined by Equation 4.4.

$$H_{\rm G}^{\rm sat} = C_P^{\rm air} T_{\rm wet} + W \left(H_{\rm vap} + C_P^{\rm vap} T_{\rm wet} \right) \tag{4.4}$$

In which:

$$C_P^{air} = 1.006 \text{ kJ} \circ \text{C}^{-1} \text{ kg}^{-1} \text{ dry air}$$

 $H_{\text{vap}} = 2501 \text{ kJ} \text{ kg}^{-1} \text{ vapour}$
 $C_P^{\text{vap}} = 1.86 \text{ kJ} \circ \text{C}^{-1} \text{ kg vapour}$

The value of $H_{G,op}$ is given by the operating line which connects the inlet and outlet conditions of the air stream (Equation 4.5).

$$H_{\rm G,op}(T_{\rm L}) = H_{\rm G,in} + \frac{L \ C_{P,\rm L}}{G} (T_{\rm L} - T_{\rm L,out})$$
(4.5)

The cooling tower fill packing has an important role in the heat and mass transfer processes by increasing the interface between the air and water flows (Lemouari et al., 2007). The mass transfer coefficient of the tower packing ($K_x a$) and the fluxes G and L can be correlated by a power law suggested by Mills (2001) (Equation 4.6). The constants C_1 , n_1 and n_2 are defined according to the packing, in which $G_0 = L_0 = 3.391 \text{ kg m}^{-2} \text{ s}^{-1}$.

$$\frac{K_x a}{L} = C_1 \left(\frac{L}{L_0}\right)^{n_1} \left(\frac{G}{G_0}\right)^{n_2} \tag{4.6}$$

The packing volume that is required for a given cooling water flowrate is calculated according to Equation 4.7.

$$V_{\text{pack}} = \frac{F_{\text{in}}^{\text{tower}}}{L} \times z_{\text{tower}}$$
(4.7)

Owing to the non-linearity of Equation 4.2 and, hence, Equations 4.3 and 4.4, numeric procedures are used to estimate the minimum gas load (G_{\min}) . However, in this study, an analytical procedure is proposed to estimate this variable by fitting a quadratic function to the equilibrium curve $(H_{\rm G}^{\rm sat})$, as described in the following section.

4.1.1 Polynomial regression for the equilibrium curve

If a large number of $H_{\rm G}^{\rm sat}$ values are calculated using Equations 4.2, 4.3 and 4.4, the equilibrium curve $H_{\rm G}^{\rm sat}$ can be plotted in an enthalpy versus temperature graph.



FIGURE 4.2: Equilibrium curve of $H_{\rm G}^{\rm sat}$ $P_{\rm atm} = 101.325 \, \rm kPa$

As can be seen in Figure 4.2, its behaviour can be approximated to a parabolic curve of a quadratic function. In order to fit a quadratic function to the equilibrium curve, three enthalpy values are necessary to obtain the parameters a_e , b_e and c_e in Equation 4.8.

$$H_{\rm G}^{\rm sat, fit} = a_e T^2 + b_e T + c_e \tag{4.8}$$

The temperature range that a cooling tower operates can provide two limiting values to fit the curve. The superior and inferior limiting enthalpies can be obtained from the water inlet temperature $(T_{\rm in}^{\rm tower})$ and outlet temperature $(T_{\rm out}^{\rm tower})$ in the cooling tower, respectively. The average water temperature in the cooling tower can be chosen as the intermediate point to fit the quadratic function.

$$T_{\rm ave} = \frac{T_{\rm L,in}^{\rm tower} + T_{\rm L,out}^{\rm tower}}{2} \tag{4.9}$$

The fitted coefficients can be calculated by the solution of a linear system and can be represented by Equation 4.10.

$$y = Ax \quad \therefore \quad x = A^{-1}y \tag{4.10}$$

In which:

$$y = \begin{pmatrix} H_{\rm G}^{\rm sat}(T_{\rm G,in}) \\ H_{\rm G}^{\rm sat}(T_{\rm ave}) \\ H_{\rm G}^{\rm sat}(T_{\rm wet}) \end{pmatrix} \quad A = \begin{pmatrix} T_{\rm G,in}^2 & T_{\rm G,in} & 1 \\ T_{\rm med}^2 & T_{\rm ave} & 1 \\ T_{\rm wet}^2 & T_{\rm wet} & 1 \end{pmatrix} \quad x = \begin{pmatrix} a_e \\ b_e \\ c_e \end{pmatrix}$$

4.1.2 Minimum airflow in a cooling tower

After fitting a quadratic function to the equilibrium curve, the minimum airflow G_{\min} can be determined, as shown in Figure 4.3. The minimum airflow is obtained when the operating line tangents the saturation curve and the outlet airflow is in equilibrium with the liquid water. In other words, this flow is determined when the subtraction of $H_{G,sat}$ and $H_{G,op}$ is zero and the cooling tower height tends to infinite.



FIGURE 4.3: Equilibrium curve and operating line for different airflows

By combining the quadratic function (Equation 4.8) with the saturation curve and operating line (Equation 4.5), the subtraction of $H_{G,sat}$ and $H_{G,op}$ can be reduced to Equation 4.11. Since this equation must be zero at just one point, there must be only one possible temperature in which the operating line tangents the equilibrium curve. Thus, the quadratic function has only one root and Equation 4.12 must be satisfied.

$$H_{\rm G,sat} - H_{\rm G,op} = aT_L^2 + bT_L + c = 0 \tag{4.11}$$

$$b^2 - 4ac = 0 \tag{4.12}$$

In which:

$$a = a_e$$

$$b = b_e - \frac{L C_{P,L}}{G}$$

$$c = c_e - \left(H_{G,in} - \frac{L C_{P,L}}{G} T_{L,out}\right)$$

$$\vdots$$

$$- \frac{L C_{P,L}}{G_{min}}\right)^2 - 4a_e \left(c_e - \left(H_{G,in} - \frac{L C_{P,L}}{G_{min}} T_{L,out}\right)\right) = 0 \qquad (4.13)$$

Calling $\frac{L C_{P,L}}{G_{\min}} = x$, Equation 4.13 can be reduced to a quadratic function whose parameters are well-known, as shown in Equation 4.14.

$$a'x^2 + b'x + c = 0 (4.14)$$

In which:

 $\left(b_e\right)$

$$a' = 1$$
$$b' = -2b_e - 4a_e T_{\text{L.out}}$$
$$c' = b_e^2 + 4a_2(H_{\text{G,in}} - c_e)$$

From the two roots of the quadratic function (Equation 4.14), the positive one is used to calculate the minimum airflow in Equation 4.15.

$$G_{\min} = \frac{L \ C_{P,L}}{x_1} \quad \Leftrightarrow \quad x_1 > 0 \tag{4.15}$$

4.1.3 Cooling tower height design

The integration of Equation 4.1 can be calculated by a numerical procedure, using the trapezoidal or Composite Simpson's rules. If the interval $[T_{\text{L,out}}, T_{\text{L,in}}]$ is split up in *n* subintervals, for *n* an even number, the Composite Simpson's rule (Equation 4.16) can be applied to estimate the cooling tower height (Equation 4.1).

$$\int_{a}^{b} f(x) \, dx \approx \frac{h}{3} \left[f(a) + f(b) + 4 \sum_{i=1}^{n/2} f(a + (2i-1)h) + 2 \sum_{i=1}^{(n-2)/2} f(a+2ih) \right]$$
(4.16)

In which:

$$h = \frac{T_{\rm L,in} + T_{\rm L,out}}{n}$$

4.1.4 Water outlet temperature in a cooling tower

The numerical procedure can be efficiently used for calculation of the cooling water outlet temperature for a specified cooling tower. For a given cooling tower geometry and operating conditions, an inverse path calculation must be done to evaluate the water outlet temperature $(T_{\text{L,out}})$.

The maximum and minimum limits for the water outlet temperature $(T_{\rm L,out})$ are determined by the cooling water inlet temperature $(T_{\rm L,in})$ and the wet-bulb temperature $(T_{\rm wet})$, respectively. In other words, there must be a value for $T_{\rm L,out}$ between the interval $]T_{\rm wet}, T_{\rm L,in}]$ that Equation 4.17 reaches the value zero.

$$F(T_{\rm L,out}) = z_{\rm real} - z_{\rm calc}(T_{\rm L,out}) = 0 \qquad T_{\rm wet} < T_{\rm L,out} \le T_{\rm L,in}$$
(4.17)

As the function $z_{\text{calc}}(T_{\text{L}})$ is continuous, the Bisection Method can be used in this problem as a root-finding algorithm (Figure 4.4). In order to find the value for T_{L} that satisfies the objective function (Equation 4.17), this method requires two initial values a and b, whose respective functions F(a) and F(b) have opposite signs. The maximum and minimum limits could be used as the two initial values to ensure the opposite signs restriction. However, to avoid the infinite value when the wet-bulb temperature (T_{wet}) is used to design a cooling tower height, a value 0.1 °C above T_{wet} is used as the minimum temperature.



FIGURE 4.4: Bisection Method as a root-finding algorithm for cooling tower height

4.2 Cooling tower performance

The cooling tower performance can be analysed by changing the inlet conditions of water and air. The variable effectiveness ε is one parameter that can be used to assess the influence of an operating condition on the cooling tower performance (Equation 4.18).

$$\varepsilon = \frac{Q}{Q_{\text{max}}} \cong \frac{T_{\text{L,in}} - T_{\text{L,out}}}{T_{\text{L,in}} - T_{\text{wet}}}$$
(4.18)

Calculating $T_{\text{out}}^{\text{tower}}$ by this method for different values of inlet temperature $(T_{\text{in}}^{\text{tower}})$ and cooling-water flow (L), the behaviour of the tower effectiveness can be analysed, as shown in Figure 4.5.



(B) Effectiveness ε

FIGURE 4.5: Sensitivity analysis in a cooling tower

As the inlet temperature increases and the cooling-water flow decreases in a cooling tower, the effectiveness rises. In this context, the cooling water reuse in heat exchanger networks may come as an alternative to increase the cooling tower performance. As the cooling water return flowrate decreases and its temperature rises, more waste heat can be rejected to the atmosphere in the cooling tower. However, practical constraints might limit the cooling water return temperature, such as temperature limits for the packing materials in the cooling tower, fouling from the cooling water and corrosion considerations in the heat exchangers and pipework (Smith, 2005).

4.3 Case study application

In this section, cooling towers are modelled for different heat exchanger networks from the previous chapters. According to the operating conditions in each network, the tower must cool the cooling water at temperature $T_{\text{tower}}^{\text{in}}$ until a specified $T_{\text{tower}}^{\text{out}}$. To evaluate the cooling tower height, the following assumptions are considered:

- A counterflow mechanical draft tower
- Wet-bulb temperature is 18 °C;
- Dry-bulb temperature is 30 °C;
- Atmospheric pressure is 101.15 kPa;
- Outlet temperature $T_{\text{tower}}^{\text{out}}$ is 20 °C;
- Gas load rate (G) at 1.5 of the minimum gas load rate G_{\min} ;
- Cooling water load rate (L) is equal to $1 \text{ kg m}^{-2} \text{ s}^{-1}$ (Albright, 2008);
- Water specific heat capacity (C_P) is 4.1816 kJ kg⁻¹ K⁻¹;
- Counterflow packing used: Flat sheets, pitch 2.54×10^{-2} m ($C_1 = 0.459$, $n_1 = -0.73$, $n_2 = 0.73$) (Mills, 2001);
- Evaporation/drift flows are negligible and makeup and blowdown flows are equal;

For the heat exchanger network in parallel arrangement, the cooling tower operating conditions are shown in Figure 4.6.



FIGURE 4.6: Cooling tower for the parallel arrangement

For each heat exchanger networks with one reuse stream, the cooling tower model is presented in Figure 4.8.



FIGURE 4.7: Cooling tower for heat exchanger networks with one reuse stream and watersaving efficiency ε of 22.2



FIGURE 4.8: Cooling tower for heat exchanger networks with one reuse stream and watersaving efficiency ε of 77.8

For two cooling water reuse streams, the cooling tower is modelled for each heat exchanger network from the previous chapter. As the networks have the same cooling water flowrate at temperature $T_{\text{tower}}^{\text{in}}$, the cooling tower is equal for the three different networks.



FIGURE 4.9: Cooling tower for heat exchanger networks with two reuse stream and water-saving efficiency ε of 100.0%

Analysing the different cooling water systems, a decrease in the cooling tower size is verified as cooling water is saved. As greater is the water-saving efficiency in the heat exchanger network, higher is the cooling tower effectiveness and lower is its required volume. For this case study, a linear dependence can be noticed between the cooling tower volume (V) and the water-saving efficiency (ε), as can be seen in Figure 4.10. This graph was created by the results from the different arrangements that are shown in Table 4.1 and also illustrated in Appendix C. Applying a linear regression for the variables V and ε , the angular coefficient indicates that the reduction of the cooling tower volume occurs in a rate of $0.7 \text{ m}^3/(\%)$ as cooling water is reused.

TABLE 4.1: Cooling tower volume for different water-saving efficiency ε

Volume (m^3)	ε
292.3	0.0
278.6	22.2
253.8	59.3
240.9	77.8
239.5	79.6
224.0	100.0



FIGURE 4.10: Effect of water saving efficiency on cooling tower volume

Chapter 5

Cooling water system design

Most cooling water systems start from a grass-root design, in which a new project is planned with large flexibility regarding plant layout. In this project, all equipment can be optimised from the beginning, before purchasing and installation (Nordman, 2005).

After a grass-root design, some plants may need a retrofit of the existing equipment to reduce the utility consumption of an existing heat exchanger network or to increase the throughput. In this case, the equipment topology plays an important role and must be considered to create a feasible design (Smith, 2005).

Both grass-root and retrofit situations are presented in the following sections to design a cooling water system. In a grass-root design, an algorithm is proposed to search the heat exchanger network that provides the minimum cooling water flowrate for different numbers of cooling water reuse streams. The impact of retroffiting a heat exchanger network on the cooling water system is also assessed.

5.1 Grass-root Design

In a grass-root project for cooling water system, both heat exchangers and cooling water tower specifications are calculated according to the hot process requirements. Facing a wide possibility of designs, the project aims to satisfy the mass and energy balances at a minimum cost.

In the present study, the methodologies to model a cooling tower and different heat exchanger networks are integrated to design a cooling water system. The algorithm of Figure 5.1 is proposed for a grass-root design to create different cooling water systems with minimum water recirculation. The heat exchangers profile specifies the heat load Q_i and the temperatures $T_{\rm in}^{\rm max}$ and $T_{\rm out}^{\rm max}$. Air properties, such as atmospheric pressure $(P^{\rm atm})$, dry-bulb $(T_{\rm dry})$ and wet-bulb $(T_{\rm wet})$ temperatures, are required to design the cooling tower. Flow velocity $(v_{\rm T})$ of the cooling water in the tubes is defined in order to evaluate the pressure drop and estimate the fouling resistance in each heat exchanger. A maximum number of cooling water reuse $(n_{\rm reuse}^{\rm max})$ can be defined to analyse heat exchangers networks in a series-parallel arrangement.





5.2 Retrofit Design

In a retrofit design, the amount of constraints imposed on the solution by the existing process layout is very large if compared to a grass-root design. In general, retrofit design aims minimum process modifications at the minimum cost. For a cooling water system, a retrofit may be necessary if a new heat exchanger is inserted into the existing heat exchanger network or the hot process throughput is increased. In both cases, the additional waste heat may bottleneck the system, compromising the existing cooling tower, heat exchanger network and/or pumping system.

The cooling water reuse may become a retrofit alternative to debottleneck a cooling water system, as studied by Kim and Smith (2003). Since recirculating water requirement is reduced as water is reused, the cooling tower can operate in a higher performance (Figure 4.5b) and reject more waste heat to the atmosphere. However, each component in a cooling water system must be analysed for the new operating condition.

Retrofitting a network from a parallel to a series-parallel arrangement may carry to heat exchangers with different operating conditions. In the retrofitted condition, the heat exchanger areas must be large enough to fulfil the heat load, as expressed by Equation 5.1.

$$A_i^{\text{retro}} \le A_i^* \tag{5.1}$$

Considering the variables Q_i , U_i and $\Delta T_{\text{LM},i}$ as the operating conditions for a given heat exchanger *i*, the heat transfer area (A_i) can be calculated by Equation 5.2.

$$A_i = \frac{Q_i}{U_i \ \Delta T_{\text{LM},i}} \tag{5.2}$$

Substituting Equation 5.2 into Equation 5.1:

$$\frac{Q_i^{\text{retro}}}{U_i^{\text{retro}} \Delta T_{\text{LM},i}^{\text{retro}}} \le \frac{Q_i^*}{U_i^* \Delta T_{\text{LM},i}^*}$$
(5.3)

If $Q_i^{\text{retro}} = Q_i^*$:

$$U_i^{\text{retro}} \ \Delta T_{\text{LM},i}^{\text{retro}} \ge U_i^* \ \Delta T_{\text{LM},i}^* \tag{5.4}$$

Or:

$$\frac{U_i^{\text{retro}}}{U_i^*} \ge \frac{\Delta T_{\text{LM},i}^*}{\Delta T_{\text{LM},i}^{\text{retro}}} \tag{5.5}$$

It is known that the overall heat transfer coefficient (U) and the log mean temperature difference $(\Delta T_{\rm LM})$ depend on the cooling water velocity and the inlet and outlet temperatures in a heat exchanger, respectively. If cooling water is reused in an existing heat exchanger, on the one hand, the utility flowrate may increase, thereby raising the cooling water velocity and, hence, its coefficient U. On the other hand, since the cooling water is supplied, partially or totally, by other heat exchanger, its inlet cooling water temperature may increase, thereby decreasing $\Delta T_{\rm LM}$. According to Equation 5.5, the increase in the coefficient U must compensate the reduction in $\Delta T_{\rm LM}$, otherwise, additional heat transfer area or even a new heat exchanger may be required. However, purchasing of additional heat transfer area must be avoided since this strategy may increase the retrofit design cost and affect its feasibility (Wang et al., 2013).

Another component that must be analysed in this retrofit design is the pumping system. The behaviour of the system and pump characteristic curves can be illustrated in Figure 5.2. The operating point is represented by the intersection between both curves and defines the cooling water flowrate F. In general, the pump is designed to work at the best condition point which is close to the best efficiency point (BEP) (Chaurette, 2001).



FIGURE 5.2: Pump and system curves representation

If the heat exchanger network is retrofitted to reuse cooling water, the critical path of the new heat exchanger network may provide a different system curve $H_{\rm s}^{\rm retro}$, as depicted in Figure 5.3.



FIGURE 5.3: New characteristic system curve after retrofitting

In Figure 5.4, two different cases are presented to analyse the relationship between the new system curve $H_{\rm s}^{\rm retro}$ and the required flowrate with cooling water reuse $F_{\rm net}^{\rm reuse}$. The existing pump can operate at the retrofitted operating condition only if Equation 5.6 is satisfied.



 $H_{\rm p}|_{F_{\rm net}^{\rm reuse}} \ge H_{\rm s}^{\rm retro}|_{F_{\rm net}^{\rm retro}}$ (5.6)

FIGURE 5.4: System and Pump Characteristic Curves

In Case 1 (Figure 5.4a), although the system characteristic curve has changed, the condition from Equation 5.6 is satisfied, i.e., the value of $H_{\rm p}$ remains higher than $H_{\rm s}^{\rm retro}$ at the desired flowrate $F_{\rm net}^{\rm reuse}$. In this case, the system characteristic curve $H_{\rm s}^{\rm retro}$ can be adjusted in order to obtain the desired cooling water flowrate $F_{\rm net}^{\rm reuse}$. As a disadvantage, the new operating point may be far from the best efficiency point (BEP) and the pump may operate at a low efficiency ($\eta_{\rm p}$). In Case 2 (Figure 5.4b), the pump head $H_{\rm p}$ is lower than the system characteristic curve $H_{\rm s}^{\rm retro}$ at the desired flowrate $F_{\rm net}^{\rm reuse}$. In this case, changing the pump or associating a new pump in series with the existing one would be necessary to increase the pump curve to $H_{\rm p}^{\rm new}$.

By including this previous analysis for a retrofit scenario, a new algorithm is proposed, as shown in Figure 5.5. Differently from the grass-root situation, the proposed algorithm searches a heat exchanger network that can debottleneck a cooling tower with minimum cooling water reuse streams. Besides of having more constraints to be satisfied, the algorithm attempts to assess the existing of previous pieces of equipment, giving high priority to expensive units (i.e., cooling tower) rather than cheap ones (i.e., pump).



FIGURE 5.5: Proposed retrofit design algorithm

Chapter 6

Conclusion and suggestions for further work

6.1 Conclusion

The present study introduced a methodology to design different cooling water system at minimum utility requirement and to analyse the impacts of cooling water reuse on the heat exchanger network pressure drop and on the cooling tower size. By using combinatorial algorithms in conjunction with a superstructure model, different heat exchanger networks could be created for a given number of heat exchanger and cooling water reuse streams. According to some network constraints, the minimum utility requirement could be achieved for each structure by solving a nonlinear programming optimisation problem in Microsoft Excel Solver. Some aspects of the heat exchanger network pressure drop and the cooling tower could also be analysed for different cooling water systems.

By applying the methodology in a case study, positive and negative aspects of different cooling water systems could be analysed. On the one side, the study has shown that some systems with cooling water reuse could reduce not only recirculating water, but also the cooling tower volume requirement. Both features may influence positively the capital and/or the operational expenditures of the cooling water system. Initially, by reducing recirculating

water, less utility can be purchased to operate the cooling water system. Additionally, by reducing the cooling tower volume requirement, few materials, including fill packing, may be required to build and operate the cooling tower.

On the other side, cooling water reuse may have negative aspects that affect the cooling water system and increase capital/operational expenditures. The study has presented that, since cooling water reuse leads to a series-parallel arrangement, the heat exchanger network pressure drop may increase and affect negatively the pumping system. Furthermore, since cooling water reuse may result in an increased temperature profile in the system, this effect may have a negative impact on heat transfer area, cooling tower packing, fouling and corrosion aspects.

However, a more detailed technical-economic analysis could be suggested for further work to analyse which cooling water system is more economically feasible. Since this analysis requires particular process details, the present study focused on proposing a methodology to give insights of different cooling water system for a generic and conceptual project design. In order to choose the most appropriate design, a feasibility study could provide important basis for decision-making during the project design.

Owing to ubiquity of Microsoft Excel in industry, the methodology has had the advantage of being able to be applied in most computers. Without requiring different optimisation software, the package Solver in Excel could be successfully used to converge at the minimum cooling water conditions for each heat exchanger structure. However, since Microsoft Excel Solver uses the Generalized Reduced Gradient for optimising nonlinear problems, the global optimal solution could not be guaranteed and, therefore, other optimisation algorithms might be used to overcome this limitation or to verify the probable globally optimal solution.

Furthermore, for large numbers of heat exchangers and/or reuse streams, a considerable number of decision variables and limiting constraints can be created, thereby exceeding the standard Solver limit. In Microsoft Excel 2013, the standard package Solver has a limit of 200 decision variables and 100 limiting constants. If these numbers are exceeded, a Premium Solver package or other optimisation software could be selected to overcome this limitation.

6.2 Suggestions for further works

The following issues merit further detailed research.

- Controllability and operability analysis of heat exchanger networks in series-parallel arrangement Although this study has detailed a procedure to evaluate the pressure drop in heat exchanger networks in series-parallel layout, this type of arrangement may be more difficult to control and operate rather than conventional parallel arrangements.
- Fouling impacts on cooling water system Fouling mechanisms are important aspects to study and consider during the cooling water system design. Since cooling water reuse may increase the temperature profile in some pieces of equipment, fouling may influence negatively the operating conditions of the system. Because of the inverse solubility of some salts in water, crystallisation and deposition of dissolved salts may contribute to the fouling mechanism in the system and the cooling water reuse may be impractical.
- Technical-economic analysis of cooling water systems a detailed technical-economic analysis can provide information of which cooling water system is more feasible for a given process. Other aspects, such as equipment costs, design complexity and operability could be analysed during a cooling water system design.
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Appendix A

Heat exchanger networks - Case Study



Arrangement 1

FIGURE A.1: Arrangement 1 - two reuse stream and water-saving efficiency ε of 100% $(F~{\rm in}~{\rm kg\,s^{-1}}$ and $Q~{\rm in}~{\rm kW})$



Arrangement 2

FIGURE A.2: Arrangement 2 - two reuse stream and water-saving efficiency ε of 100% $(F~{\rm in}~{\rm kg\,s^{-1}}$ and $Q~{\rm in}~{\rm kW})$



Arrangement 3

FIGURE A.3: Arrangement 3 - two reuse stream and water-saving efficiency ε of 100% $(F~{\rm in}~{\rm kg\,s^{-1}}$ and $Q~{\rm in}~{\rm kW})$



Arrangement 4

FIGURE A.4: Arrangement 4 - two reuse stream and water-saving efficiency ε of 79.6% (F in kg s⁻¹ and Q in kW)



Arrangement 5

FIGURE A.5: Arrangement 5 - two reuse stream and water-saving efficiency ε of 77.8% $(F~{\rm in}~{\rm kg\,s^{-1}}$ and $Q~{\rm in}~{\rm kW})$



Arrangement 6

FIGURE A.6: Arrangement 6 - two reuse stream and water-saving efficiency ε of 59.3% $(F~{\rm in}~{\rm kg\,s^{-1}}~{\rm and}~Q~{\rm in}~{\rm kW})$



Arrangement 7

FIGURE A.7: Arrangement 7 - two reuse stream and water-saving efficiency ε of 22.2% $(F~{\rm in}~{\rm kg\,s^{-1}}~{\rm and}~Q~{\rm in}~{\rm kW})$

Appendix B

Pressure Drop in cooling water network - Case study



Arrangement 1

FIGURE B.1: Arrangement 1 - two reuse stream and water-saving efficiency ε of 100.0% (*P* in kPa and *Q* in kW)



Arrangement 2

FIGURE B.2: Arrangement 2 - two reuse stream and water-saving efficiency ε of 100.0% (*P* in kPa and *Q* in kW)



Arrangement 3

FIGURE B.3: Arrangement 3 - two reuse stream and water-saving efficiency ε of 100.0% (*P* in kPa and *Q* in kW)



FIGURE B.4: Arrangement 4 - two reuse stream and water-saving efficiency ε of 79.6% (*P* in kPa and *Q* in kW)



Arrangement 5

FIGURE B.5: Arrangement 5 - two reuse stream and water-saving efficiency ε of 77.8% (P in kPa and Q in kW)



Arrangement 6

FIGURE B.6: Arrangement 7 - two reuse stream and water-saving efficiency ε of 59.3% (P in kPa and Q in kW)



FIGURE B.7: Arrangement with two reuse stream and water-saving efficiency ε of 22.2% (*P* in kPa and *Q* in kW)

Appendix C

Cooling towers and the cooling water network - Case Study



Arrangement 1

FIGURE C.1: Arrangement 1 - cooling tower for two reuse stream and water-saving efficiency ε of 100.0% (*P* in kPa and *Q* in kW)



FIGURE C.2: Arrangement 2 - cooling tower for two reuse stream and water-saving efficiency ε of 100.0% (*P* in kPa and *Q* in kW)



FIGURE C.3: Arrangement 3 - cooling tower for two reuse stream and water-saving efficiency ε of 100.0% $(P~{\rm in~kPa}~{\rm and}~Q~{\rm in~kW})$



Arrangement 4

FIGURE C.4: Arrangement 4 - cooling tower for two reuse stream and water-saving efficiency ε of 79.6% (*P* in kPa and *Q* in kW)



Arrangement 5

FIGURE C.5: Arrangement 5 - cooling tower for two reuse stream and water-saving efficiency ε of 77.8% (P in kPa and Q in kW)



Arrangement 6

FIGURE C.6: Arrangement 6 - cooling tower for two reuse stream and water-saving efficiency ε of 59.3% (*P* in kPa and *Q* in kW)



FIGURE C.7: Arrangement 7 - cooling tower for two reuse stream and water-saving efficiency ε of 22.2% $(P~{\rm in~kPa}~{\rm and}~Q~{\rm in~kW})$